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ENGINEERING DESIGN HANDBOOK

FIRE CONTROL SERIES

COMPENSATING ELEMENTS

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HEADQUARTERS, U. S. ARMY MATERIEL COMMAND SEPTEMBER 1963

ENGINEERING DESIGN HANDBOOK SERIES

Listed below are the Handbooks which have been published or submitted for publication. Handbooks with publication dates prior to 1 August 1962 were published as 20-series Ordnance Corps pamphlets. AMC Circular 310-38, 19 July 1963, redesignated these publications as 706-series AMC pamphlets (i.e., ORDP 20-138 was redesignated AMCP 706-138). All new, reprinted, or revised Handbooks are being published as 706-series AMC pamphlets.

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106	Elements of Armament Engineering, Part One, Sources of Energy
107	Elements of Armament Engineering, Part Two, Ballistics
108	Elements of Armament Engineering, Part Three, Weapon Systems and Components
110	Experimental Statistics, Section 1, Basic Concepts and Analysis of Measurement Data
111	Experimental Statistics, Section 2, Analysis of Enumerative and Classificatory Data
112	Experimental Statistics, Section 3, Planning and Analysis of Comparative Experiments
113	Experimental Statistics, Section 4, Special Topics
114	Experimental Statistics, Section 5, Tables
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134	Maintenance Engineering Guide for Ordnance Design
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136	Servomechanisms, Section 1, Theory
137	Servomechanisms, Section 2, Measurement and Signal Converters
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139	Servomechanisms, Section 4, Power Elements and System Design
170(C)	Armor and Its Application to Vehicles (U)
250	Guns--General (Guns Series)
252	Gun Tubes (Guns Series)
270	Propellant Actuated Devices
290(C)	Warheads--General (U)
331	Compensating Elements (Fire Control Series)
355	The Automotive Assembly (Automotive Series) (Revised)

Ammunition and Explosives Series

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176(C)	Solid Propellants, Part Two (U)
177	Properties of Explosives of Military Interest, Section 1
178(C)	Properties of Explosives of Military Interest, Section 2 (U)
179	Explosive Trains
210	Fuzes, General and Mechanical
211(C)	Fuzes, Proximity, Electrical, Part One (U)
212(S)	Fuzes, Proximity, Electrical, Part Two (U)
213(S)	Fuzes, Proximity, Electrical, Part Three (U)
214(S)	Fuzes, Proximity, Electrical, Part Four (U)
215(C)	Fuzes, Proximity, Electrical, Part Five (U)
244	Section 1, Artillery Ammunition--General, with Table of Contents, Glossary and Index for Series
245(C)	Section 2, Design for Terminal Effects (U)
246	Section 3, Design for Control of Flight Characteristics
247	Section 4, Design for Projection
248	Section 5, Inspection Aspects of Artillery Ammunition Design
249	Section 6, Manufacture of Metallic Components of Artillery Ammunition

Ballistic Missile Series

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150	Interior Ballistics of Guns
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HEADQUARTERS
UNITED STATES ARMY MATERIEL COMMAND
WASHINGTON 25, D.C.

30 September 1963

AMCP 706-331, Compensating Elements, forming part of the Fire Control Series of the Army Materiel Command Engineering Design Handbook Series, is published for the information and guidance of all concerned.

(AMCRD)

FOR THE COMMANDER:

SELWYN D. SMITH, JR.
Major General, USA
Chief of Staff

OFFICIAL:

R. O. Davidson

R. O. DAVIDSON
Colonel, GS
Chief, Administrative Office

DISTRIBUTION: Special

PREFACE

The Engineering Design Handbook Series of the Army Materiel Command is a coordinated series of handbooks containing basic information and fundamental data useful in the design and development of Army materiel and systems. The handbooks are authoritative reference books of practical information and quantitative facts helpful in the design and development of Army materiel so that it will meet the tactical and the technical needs of the Armed Forces.

This Handbook, *Compensating Elements*, AMCP 706-331, has been prepared as one of a series on Fire Control Systems. It presents information on the effects of out-of-level conditions and displacement between a weapon and its aiming device. It also presents information on the instrumentation necessary for correction of the resulting errors, standard design practices, and considerations of general design, manufacture, and field use and maintenance.

This Handbook was prepared under the direction of the Engineering Handbook Office, Duke University, under contract to the Army Research Office-Durham. The text and illustrations were prepared by Ford Instrument Company, a division of the Sperry Rand Corporation, under subcontract to the Engineering Handbook Office. Technical assistance was rendered by Frankford Arsenal.

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Comments and suggestions on this handbook are welcome and should be addressed to Army Research Office-Durham, Box CM, Duke Station, Durham, North Carolina 27706.

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Section I
INTRODUCTION

1. General

Two of the factors that affect the laying of a weapon are the levelness of the weapon and the displacement between the weapon and its aiming device. Since the azimuth and elevation information for laying a weapon is usually based on a level reference, any out-of-level condition in a weapon system will introduce an error. The displacement between a weapon and its aiming device introduces the problem of parallax which also causes errors in weapon laying. Where the characteristics desired for a given weapon will not tolerate such errors, the effects of out-of-level conditions or parallax can be compensated. The subject of this handbook encompasses the design of instruments that will correct the laying of a weapon for errors caused by out-of-level and parallax conditions.

2. Out-of-Level Conditions

a. For a given firing situation there is only one correct azimuth and elevation setting of a weapon's bore. These values express the position of the weapon bore with respect to a level coordinate system that is referenced to a chosen compass direction. A weapon usually has indicators to show the azimuth direction and elevation of its bore with respect to its supporting frame. If azimuth and elevation in the level coordinate system are applied to the weapon through the use of the indicators, the weapon bore will be positioned properly, provided the weapon was correctly oriented and leveled. Correct orientation references the azimuth indicator to the same compass direction as the aiming device while leveling fixes the weapon traverse axis vertical and the elevation or trunnion axis horizontal. Figure 1 illustrates azimuth and elevation for a leveled weapon that has been referenced to north. (The elevation angle shown is also called quadrant elevation—the vertical angle between the line of elevation and the horizontal.) As long as the proper relationship of the trunnion and traverse axes to the horizontal reference plane exists, no azimuth or elevation compensation is required.

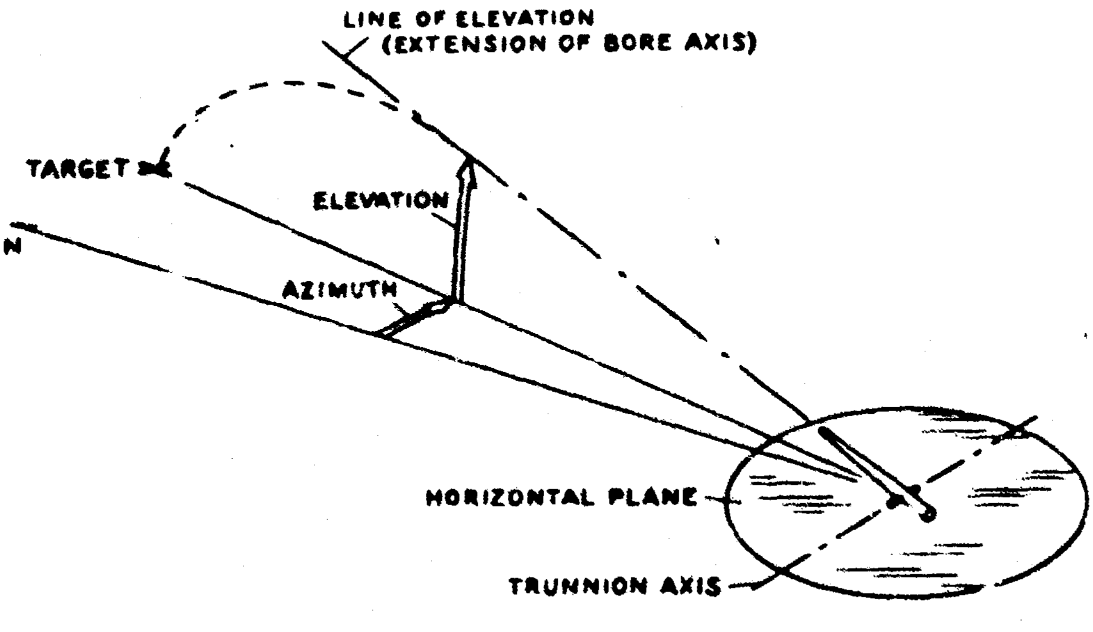


Figure 1. *Laying information for a leveled weapon*

b. However, if the weapon was not properly oriented in a horizontal reference plane, the weapon bore will not be positioned at the azimuth and elevation values shown on the indicators. Figure 2 illustrates a weapon that was initially oriented in a tilted plane instead of a horizontal plane. When the weapon was layed, the azimuth and elevation used were based on a horizontal plane. Since the weapon was positioned with reference to the tilted plane, azimuth and elevation errors exist in the position of the weapon bore.

c. To position the weapon properly so that the azimuth and elevation errors introduced by the tilted plane are eliminated, it is necessary to correct the laying information. Figure 3 illustrates some of the angles that are used in determining the amount of compensation required for correcting laying information. Quadrant elevation is always meas ed in a vertical plane from the horizontal, while gun elevation is always measured perpendicular to the plane in which the weapon is oriented, i.e., the deck plane. If the deck plane of the weapon is horizontal, the gun elevation and quadrant elevation will coincide. Azimuth is always measured from north or from some other convenient reference direction, in a horizontal plane, to a vertical plane through the weapon bore axis. The out-of-level condition of a weapon is usually expressed in terms of two components: pitch angle and cant angle. The pitch angle is the angular deviation of the weapon fore-and-aft axis from the horizontal, while the cant angle is the angular deviation of the weapon cross axis (trunnions) from the horizontal. Pitch and cant are always 90 degrees displaced from each other in the deck plane. The out-of-level condition can also be expressed in terms of one tilt angle in a vertical plane (Figure 2) and the azimuth angle of the vertical plane. This method is more fully described later. Other means can be devised for expressing a weapon's out-of-level condition, depending on the method chosen for applying corrections and the type of weapon system. Out-of-level correcting mechanisms have been built using the two approaches given above.

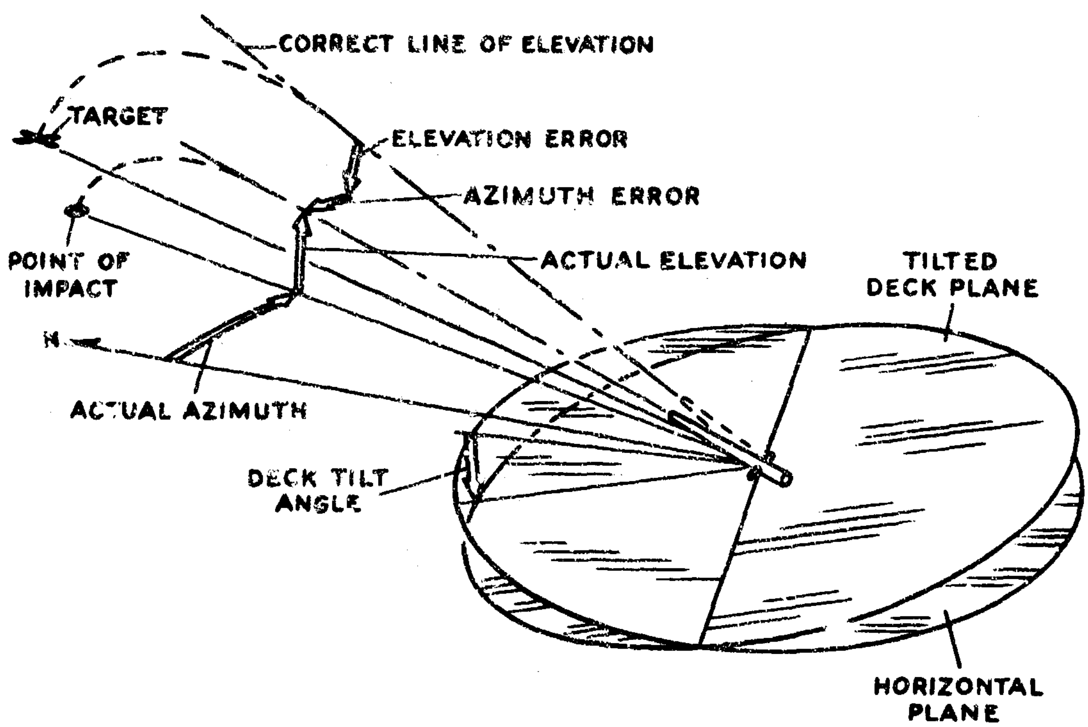


Figure 2. *Laying errors caused by cant and pitch of an out-of-level weapon*

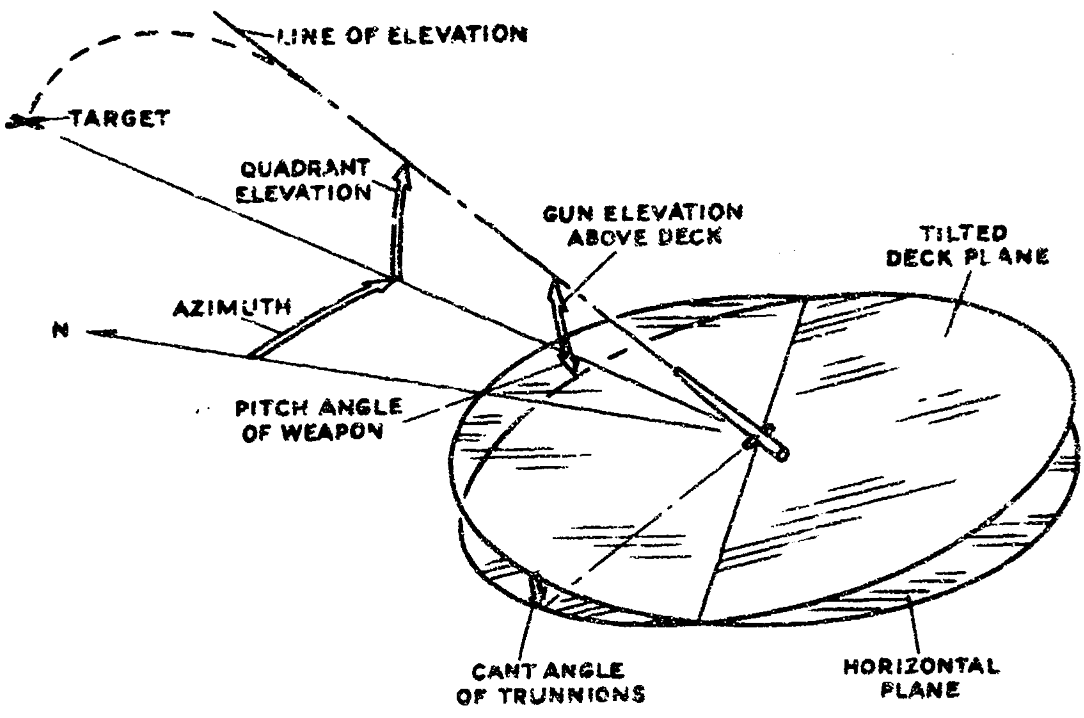


Figure 3. *Out-of-level weapon compensated for cant and pitch*

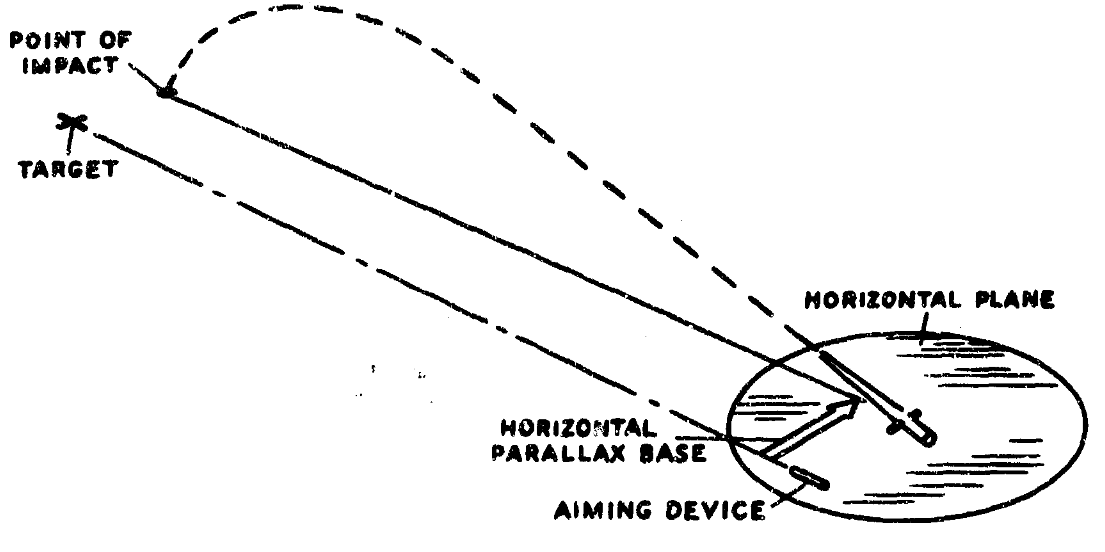


Figure 4. *Parallax caused by aiming device displacement*

3. PARALLAX

a. Parallax can be defined as the apparent difference in the position of a target as viewed from two different points—the weapon bore and aiming device. The aiming device for a weapon can be located very close to the weapon tube as in the case of a weapon-mounted sight, or at a distance as in the case of a data-gathering director of an antiaircraft installation. The term "aiming device" as used here describes any mechanism or instrument, optical, mechanical, or electrical, that is used for determining target position. An aiming device can be a simple optical sight for a field artillery weapon or part of a complicated data-gathering director for an antiaircraft weapon system. (Aiming devices should not be confused with auxiliary fire control equipment such as spotting binoculars which have no direct function in positioning the weapon bore.) Regardless of the location of the aiming device, it cannot occupy the same space that the weapon bore occupies. As a result, an error is introduced whenever an aiming device is used to lay the weapon on a target. This error is called parallax error.

b. Figure 4 illustrates parallax caused by the horizontal displacement of an aiming device. (A sight is illustrated but it might also be a data-gathering director located many yards from the weapon.) When the aiming device is sighted on a target, the weapon will not be aligned on the same point, causing the firing error shown. Parallax errors can be introduced into a system by displacement of the aiming device in any direction—horizontally or vertically. Figure 5 illustrates an aiming device that has been displaced in three directions.

c. As stated before, parallax errors are caused by the displacement between the aiming device and weapon. The parallax errors in weapon systems also are affected by the method used for initial alignment between the aiming device and weapon. Initial alignment is usually obtained by one of two methods. By one method, the infinity boresighting method, the aiming device and the weapon are aligned on some celestial body or other distant object. This method of boresighting places the axis of the weapon (gun bore) and the aiming device (line of site) essentially parallel to each other. Figures 4 and 5 show weapon systems aligned in this manner. By the other method, specific-range boresighting, the aiming device and weapon are sighted on a point at a distance, within the firing

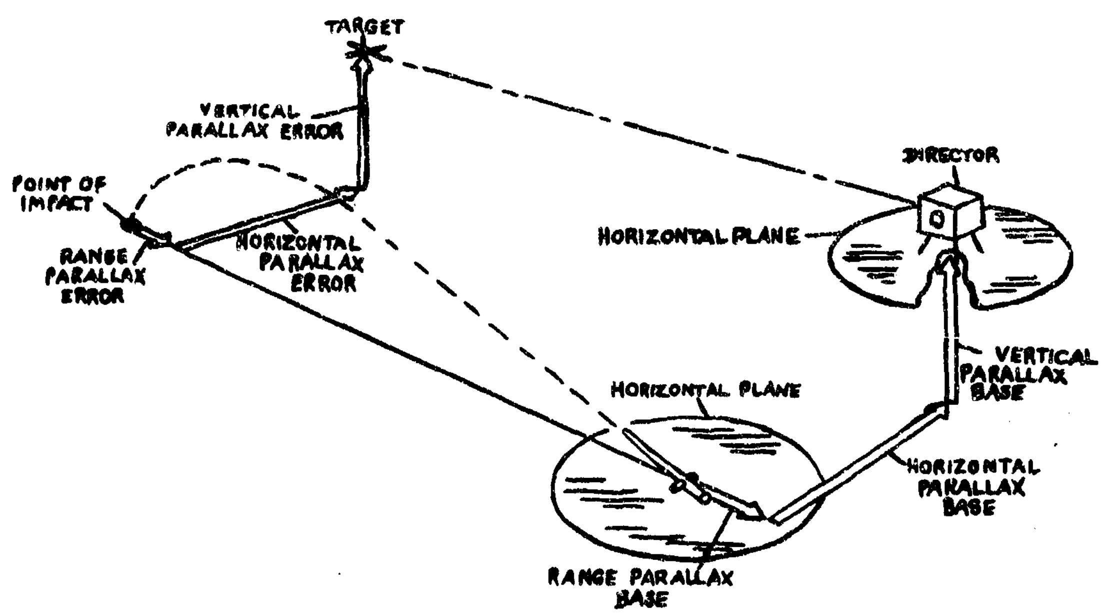


Figure 5. *Parallax error caused by displacement in three directions*

range for which the weapon is to be used. For example, boresighting for tanks is currently being done at 1500 yards. Figure 6 illustrates parallax in a specific-range boresighted system.

d. In an infinity-boresighted system, a parallax error will exist for all firing ranges of a weapon. Specific-range boresighting is often used to minimize parallax errors where full compensation is not required. However, a parallax error will exist for all firing ranges of a weapon in a specific-range boresighted system except for the range at which the weapon and aiming device were aligned. Corrections for parallax errors can be expressed in terms of an angle (azimuth and/or elevation) and target range or in terms of the displacement between aiming device and weapon. The method used depends on the weapon system and firing problem.

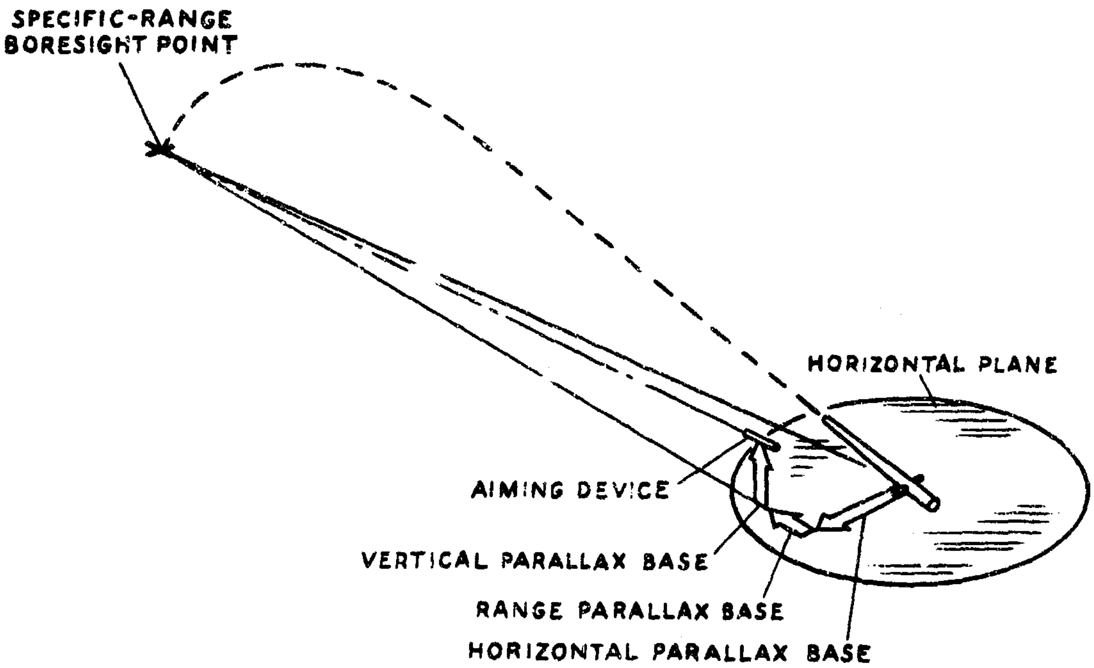


Figure 6. *Parallax in a specific-range boresighted system*

Section II

GENERAL DISCUSSION OF THE PROBLEM

4. Types of Compensation

a. Compensation for Out-of-Levelness. There are two general types of compensation that can be applied to weapons for out-of-level conditions. The type of compensation to be applied depends on whether the firing data are determined in the coordinate system of the weapon (on-carriage) or in some coordinate system other than the weapon's (off-carriage).

(1) On-Carriage Firing Data. To understand the type of compensation required for firing data determined on-carriage, it is necessary to examine the data that make up the final azimuth and elevation angles for positioning the weapon bore. Each (azimuth and elevation) provides a basic component required to align the weapon bore on the target, accounting for the direction of the target with respect to the weapon. The remaining components of the firing data consist of corrections that are applied to the weapon bore to allow for target motion and natural deviations of the projectile's path from the line of site. An example of the latter is superelevation, the ballistic correction for the drop caused by gravity. Applying superelevation to the weapon elevates it vertically above the amount required to make the bore axis intersect the target. The magnitude of superelevation varies with range and elevation in accordance with empirical (proving ground) ballistic data for the particular type of weapon and ammunition.

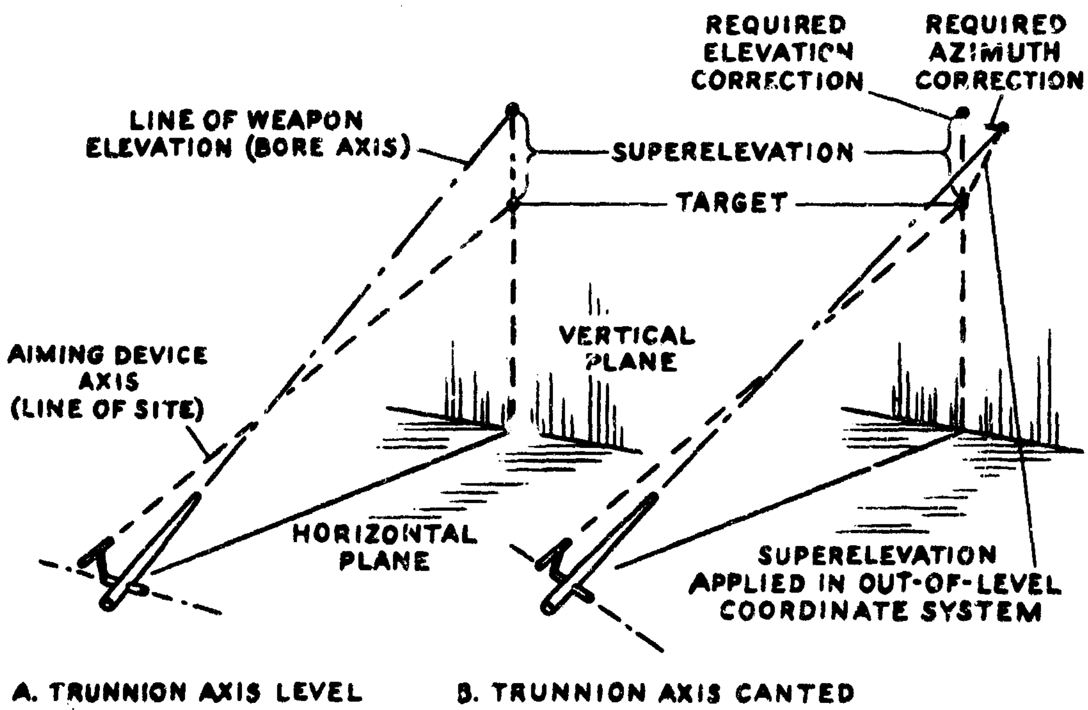


Figure 7. *On-carriage fire control — showing compensation required to transform superelevation to level coordinate system*

That portion of the firing data that aligns the weapon bore axis on the target is determined by an aiming device operating essentially in the same coordinate system as the weapon. (See Figure 7.) View A shows a weapon whose trunnions are level. The aiming device and weapon have been elevated vertically about the level trunnion axis to align on the target; and then the weapon was elevated an additional amount to compensate for gravity drop. If the weapon is tilted, the aiming device and bore tilt equally. View B of Figure 7 shows the same weapon with canted trunnions after the necessary elevation and azimuth corrections were made to realign the aiming device on the target. Realigning the aiming device moves the weapon bore a corresponding amount and if there were no superelevation, no additional compensation would be needed. But, superelevation is needed and since it is derived in a level coordinate system (gravity operates vertically), it cannot be applied simply as a direct addition to elevation when the trunnions are canted. The heavy lines in Figure 7B represent the azimuth correction and the additional elevation required to transform superelevation into the canted weapon cordinate system. In on-carriage fire control, this principle will apply to any other firing corrections that are normally referenced to a level coordinate system.

(2) Off-Carriage Firing Data. When the firing information for a weapon is determined by off-carriage equipment, it is based on the coordinate system of the off-carriage equipment and not on that of the weapon. If both coordinate systems are level, the azimuth and elevation firing information can be transferred directly from the off-carriage equipment to the gun, correcting only for parallax error. (See Figure 8A).

However, in some weapon systems it has been found impractical to make both coordinate systems level. In these systems, an out-of-level condition of the weapon, as shown in Figure 8B, will cause it to operate in an out-of-level coordinate system that is not parallel to the coordinate system of the off-carriage fire control equipment. The target location data as well as the ballistic data are based on a level coordinate system. All component parts of the firing azimuth and elevation therefore must be transformed into the weapon's coordinate system before application to the weapon.

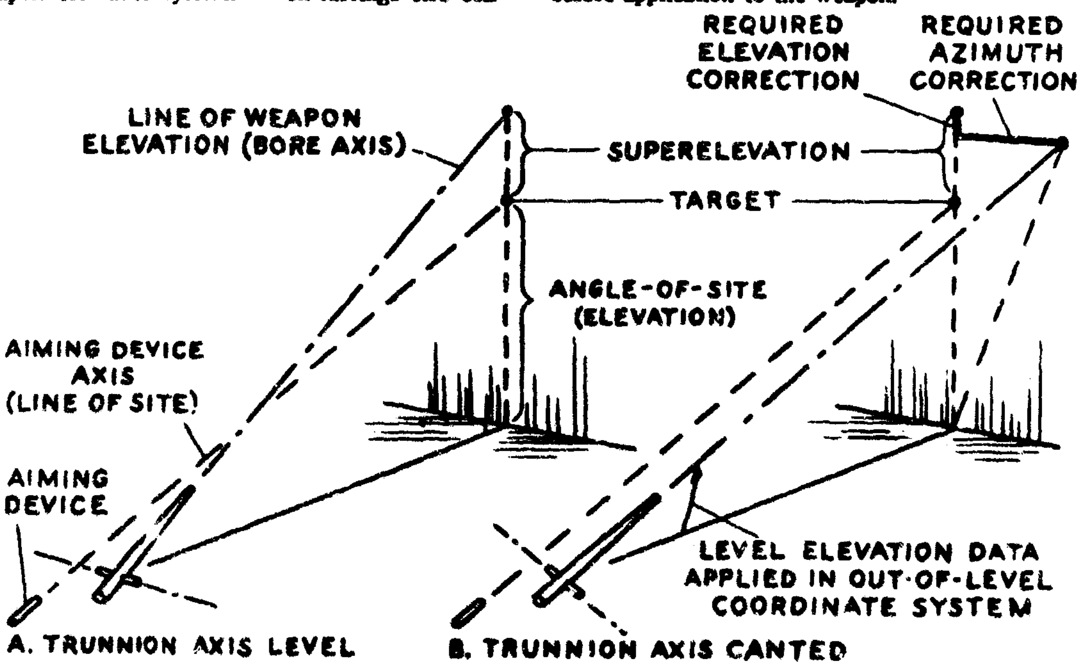


Figure 8. *Off-carriage fire control—showing compensation required to transform total elevation data to level coordinate system*

b. Compensation for Parallax. The type of compensation applied to weapons for correcting parallax errors is related to the method used for expressing the parallax error. There are three types of parallax errors that can occur in a system: horizontal, vertical, and range parallax errors. (See Figure 5.) The parallax error is usually measured in a coordinate system that is compatible with and expressed in terms that are readily combined with other data of the system. If the target's position is expressed in rectangular coordinates at any phase during computation of weapon azimuth and elevation, then the parallax displacement between weapon and aiming device also may be measured in rectangular coordinates. The compensation required would merely involve combining the displacement coordinates with the target position coordinates, thereby transforming the aiming device coordinate system to the position of the weapon. If the target's position is determined in polar coordinates and all computations leading to weapon azimuth and elevation are based on polar coordinates, then parallax compensation may be derived as angular corrections to azimuth and elevation.

5. Typical Weapon Systems

The following paragraphs give a discussion of the type of compensation that has been used in typical, existing weapon systems. Generally speaking, the compensation used in these systems varies with the tactical purpose of the weapon and the type of fire control system employed.

a. Direct Fire Weapons. A weapon is being used for direct fire when the target that it is firing upon is in the line of site of the weapon and the sighting system. The type of compensation required for such weapons depends on which category (on-carriage or off-carriage) the weapon fits into.

Some direct fire weapons, in their primary mode of operation, are designed to use firing data determined on-carriage in the coordinate system of the weapon. Others normally depend on information determined in a displaced coordinate system (off-carriage). Antiaircraft and tank weapons are typical examples of direct fire applications. Field artillery weapons are occasionally employed for direct fire though they are primarily designed for indirect fire.

(1) Antiaircraft Weapons. Antiaircraft weapons with on-carriage fire control are generally designed for a greater degree of mobility and quicker emplacement making it desirable to eliminate weapon leveling involving ground preparation, jacking, etc. Some form of compensation is required to achieve accuracy while the weapon is in an out-of-level condition.

Because this type of weapon determines target data in its own coordinate system, only the ballistic corrections applied to the basic azimuth and elevation positioning values require compensation. Often the required out-of-level corrections are applied in combination with other corrections such as those required to compensate for wind.

In on-carriage fire control, the parallax errors caused by the displacement between the antiaircraft weapon and aiming device are small when compared with the size of the target. Hence, corrections for parallax are usually ignored.

Since antiaircraft weapons of the off-carriage fire control type are located fairly permanently, they are usually leveled, thereby eliminating the need for out-of-level compensation. However, if either the aiming device or weapon, or both, cannot be leveled, compensation must be provided for the out-of-level condition. Present systems usually require that at least one (aiming device or weapon) be leveled. Out-of-levelness in these systems necessitates that the entire azimuth and elevation values used in aiming the weapon be corrected.

The displacement between the weapons and aiming device of an off-carriage fire control type antiaircraft system is usually great enough to introduce large parallax errors. However, it is sometimes desirable to ignore or only partially correct them in order to obtain definite dispersion for a battery of weapons.

(2) Tanks. Tanks are used primarily for direct fire with on-carriage fire control. Tanks usually employ high-velocity projectiles at relatively short range. The resultant trajectory is fairly flat so that the required ballistic corrections are relatively small. With the firing data determined on-carriage in the coordinate system of the gun, only the super-elevation correction for gravity drop requires compensation for out-of-level conditions. In a typical

firing procedure, the firing range is determined so that the ..ct amount of superelevation can be introduced. Out-of-level compensation may be accomplished by rotating the reticle of the sight so it is vertical. Superelevation then can be inserted in a vertical direction (See Figure 7). Out-of-level compensation also may be accomplished by a cant-correcting device that senses the amount of cant and computes the required lateral and ve... corrections to be applied to the wea... bore. Using these methods, accurate firing is confined to the condition of a stationary tank. A tank that is firing while in motion has constantly changing out-of-level conditions, especially while travelling over rough terrain. Gyrostabilizing methods have been used to compensate the changing out-of-level conditions of the moving tank. These systems have the problem of maintaining a desired weapon position despite the rapid and erratic movements of a tank.

On a tank, parallax displacement between aiming device and weapon is small. As a result, parallax errors are usually not compensated but often are minimized by specific-range boresighting techniques.

(3) Field Artillery. Direct fire is not the primary mission of field artillery weapons although they are occasionally used ... direct fire engagements. When used for direct fire, target location data is determined on-carriage in the coordinate system of the weapon. The cant compensation problem is therefore the same as discussed in the previous paragraphs on tanks. Typical compensated direct fire aiming devices for field artillery provide a means for adjusting and maintaining the elevation (range) reticle in the vertical position so that superelevation can be inserted correctly.

The location of on-carriage fire control equipment for field artillery weapons used in direct fire introduces small parallax errors that are usually ignored or minimized by specific-range boresighting.

b. Indirect Fire Weapons. Indirect fire is primarily characterized by the condition in which the target does not lie directly in the line of sight or is not visible to the gunner. Weapons with a primary mission of indirect fire have their firing information determined in a level coordinate system by off-carriage equipment. The firing information is applied to the weapon through on-carriage fire control equipment. The on-carriage fire control equipment corrects the firing information for any out-of-level condition of the gun.

(1) Field Artillery Weapons. Indirect fire is the primary mission of field artillery weapons. Since the longer ranges and higher superelevations required in indirect fire increase the errors introduced by an out-of-level condition, compensation is necessary in order to achieve accuracy. Indirect fire requires the use of target location data from an off-carriage course (usually level). The on-carriage fire control equipment must be able to transform both the off-carriage target location data and the ballistic data into the out-of-level coordinate system of the weapon, as explained in paragraph 4.a.(2) and Figure 8.

The parallax errors introduced by the on-carriage fire control equipment of field artillery are small enough to be ignored. However, if several pieces are combined to form a battery so as to direct fire at the same target, a significant parallax error will be introduced by the displacement between weapons in the battery. The desirability for correcting the parallax errors depends on the distribution of fire wanted at the target area. In some cases area fire is required, while in others, converging fire of all weapons on a relatively small target is required. Usually battery parallax errors are corrected by the battery executive when the individual guns are being zeroed-in. For this procedure, no parallax compensating device or equipment is normally provided. The corrections are determined manually on a plotting board.

(2) Tanks. Tanks are primarily designed as complete, self-contained, direct fire weapons and as such are not provided with cant compensation for indirect fire. In indirect fire they function as secondary weapons because the on-carriage fire control instruments are not designed to transform indirect firing data from an off-carriage source to the weapon coordinate system. For example, the azimuth indicator of a tank is driven directly by the turret without compensation and indicates turret traverse rather than true azimuth. If the tank is out-of-level and a value of true azimuth is applied to the azimuth indicator by traversing the turret, the azimuth of the weapon bore will not

correspond to the true azimuth. The lack of complete compensation for firing data determined off-carriage makes the problem of directing gun fire to the target more difficult and time consuming. Indirect fire therefore cannot be conducted as efficiently by tank weapons as by the field artillery weapons designed for the purpose.

6. Methods of Determining Required Compensation

a. General. Although it is possible in the less complex problems to design an analog-type solver without a preliminary mathematical analysis, it is usually more desirable to begin by obtaining a mathematical expression for the problem. Then, the mathematical expression can be instrumented by using either simple computing mechanism techniques or the more complicated computing techniques involving electrical or electromechanical elements. The solution obtained from a compensating element when an exact equation is instrumented will be a true solution. When the exact equation has been altered from true solution form for simplification, instrumentation will produce an approximate solution. The technique used for instrumenting the mathematical expression will depend on the accuracy of compensation desired and the simplicity and reliability demanded of the system.

b. True Solutions. When a mathematical analysis is used to solve a compensation problem and a true solution is desired, an exact mathematical equation must be employed. When an exact mathematical equation is instrumented, the only errors that can appear in the instrument output will be the result of manufacturing tolerances or system installation alignment and operational errors. These errors in a true solution type of instrument are referred to as "Class A" errors.

c. Approximate Solutions. The requirements of the system in which the compensation is needed may make the derivation or instrumentation of an exact equation impracticable. When this situation arises, the use of a simplified or approximated version of the exact equation is indicated. Some of the specific conditions under which approximations are considered are:

(1) The general tolerance level for other key components of the weapon does not warrant extreme accuracy in compensation.

(2) The mathematical expression is highly complex.

(3) Functions in the expression have excessive limits (Such as, tan $90° = \infty$).

(4) There is insufficient space or weight allowance for instrumentation of an exact equation.

(5) The reliability and ruggedness requirement are too stringent for complex instrumentation.

(6) Operating and maintenance techniques are overly complicated.

Under these conditions, an exact equation can be modified or approximated in such a way that it will be simplified while still maintaining the accuracy desired in the system. Some of the techniques that are commonly used when modifying or approximating equations are:

(1) Dropping high order power terms in denominators.

(2) Substituting sine functions for tangent functions of small angles.

(3) Substituting angles (in radians) for sine functions of small angles.

(4) Substituting unity for cosine functions of small angles.

(5) Using curve fitting techniques.

When a modified or approximated equation is instrumented, the answer will be an approximate solution. The errors found in an approximate solution stem from two sources. One source is the basic assumptions made during modification, simplification, or approximation of an exact equation. This type of error is commonly called a "Class B" error. The other source is from the manufacture and installation processes wherein normal tolerancing results in the "Class A" errors found in all instruments.

7. Methods of Applying Compensation

Several methods have been employed successfully for obtaining true and approximate solutions and manually or automatically applying the resultant corrections to weapon positioning quantities. Some of these methods are described below and mathematically analyzed later in the book. The methods given are not meant to be a complete list ... merely a brief description of some typical examples. Out-of-level conditions are discussed in terms of cant and pitch corrections.

a. Cant Correction (direct fire). To correct for cant in direct fire, it is necessary to assure that the required superelevation is always inserted in a true vertical direction. (Refer to paragraph 4, for explanation.) In direct fire systems where superelevation is applied by displacing the axis of an optical sight in accordance with the range graduations of the sight reticle, cant correction can be achieved by rotating the reticle to return it to the level condition. The range scale will then be made vertical enabling superelevation to be inserted without error. One method used provides both automatic and manual facilities for maintaining the sight reticle in a leveled condition. A damped pendulum serves as a vertical reference for aligning the reticle. In automatic operation, the alignment error is used in an electromechanical servo loop to control alignment corrections.

The more complex ground and antiaircraft direct-fire systems include instrumentation for computing cant corrections from the measured cant and other pertinent angles. The computed corrections are combined with other quantities pertaining to target location, lead angles, ballistic corrections, etc., to form composite azimuth and composite elevation signals. The composite signals are then fed into their respective gun positioning servo loops for automatically controlling the gun.

b. Cant Correction (indirect fire). Provision may be included in the design of on-carriage fire control equipment to correct for azimuth and elevation error caused by trunnion cant. The compensating telescope mount used with panoramic telescopes for field artillery is such a device. The compensating telescope mount is based on the operation of a simple mechanism that is essentially a Hooke's type universal joint. A Hooke's joint operates in a manner similar to a gimbal system. The input shaft of the joint is positioned by the weapon trunnion and the output shaft is used to position the panoramic telescope. (See Figure 34 and refer to paragraph 28 for complete details.) The telescope is layed on an aiming point and manually leveled, making it operate in a level coordinate system. The telescope is maintained in the level coordinate system by adjusting its mount during laying procedures. When the firing information is set in, the panoramic telescope will be thrown off its target or aiming point. The weapon is then moved to bring the panoramic telescope back on its aiming point. Because of the connection between the weapon and telescope, through the Hooke's joint in the telescope mount, the azimuth and elevation error introduced when cant is present will be automatically corrected during the realigning procedure as long as the telescope mount is maintained level.

c. Pitch Correction. The compensating telescope mount described in the previous paragraph b. also has the ability to correct for the error introduced when a weapon is pitched. As described before, the mount is connected to the weapon trunnion through a pivoted joint that allows the mount to be leveled in the fore-and-aft and cross directions. By using leveling vials to establish a horizontal reference, the mount corrects pitch errors that might be present in the weapon and allows elevation information from an off-carriage source to be inserted in a vertical direction. Pitch compensation is also provided in the design of some elevation or range quadrant mounts. Elevation and range quadrant mounts are similar, the principal difference being that one is calibrated in angles of elevation and other is calibrated in distance (range). In certain cases, quadrant mounts are calibrated in both range and elevation. These mounts are used in indirect fire for setting the weapon in elevation when this function has not been designed into the panoramic telescope mount. The range or elevation quadrant mount is also connected to the weapon trunnion through a pivoted joint. Level vials are provided on the mount so that a horizontal reference can be established readily.

d. Parallax Correction.

(1) One antiaircraft system that uses off-carriage fire control and corrects for parallax errors consists of a data-gathering type of aiming device and several weapons arranged to form a battery. The data-gathering aiming device computes some of its firing data in an orthogonal rectilinear coordinate system. Since the weapons in the battery and the data-gathering aiming device are displaced

from each other, each operates in a coordinate system of its own. The displacement between these coordinate systems is a constant distance for a given emplacement. To correct for the parallax error, it is only necessary to transform the individual coordinate systems of the weapons to the coordinate system in which the firing data is computed. The transformation is accomplished by inserting the displacement constants between the coordinate systems into the firing data through potentiometers located on the data-gathering aiming device. The computation of firing data then will be corrected for the displacement that introduced the parallax error.

(2) Systems in which data gathering and computation of firing data are conducted entirely with polar coordinates use polar coordinates also in the formulation of parallax corrections. In these systems the displacement between the director and a given gun is measured in terms of a vertical component, a horizontal component, and the azimuth angle of the horizontal component. Using the polar coordinates of target position and the measured displacement coordinates, corrections are computed for the azimuth, elevation, and range gun laying data.

(3) A method that is used for parallax correction between weapons, in the case of a field artillery battery, is applying individual parallax corrections to firing data transmitted to the weapon. The magnitudes of the corrections are obtained by estimates or measurements in conjunction with plotting operations without the benefit of special parallax-compensating mechanisms.

Section III

DETERMINATION OF REQUIRED COMPENSATION

8. General

To simplify the analyses and solutions, and to facilitate discussion of compensation problems, it is convenient to establish certain notational symbols, definitions, and reference coordinate systems. This information is presented in the following paragraphs. A large part of the discussion will be familiar to readers with a knowledge of fire control design problems; however, the reader with a limited knowledge will find the basic information presented helpful.

9. Definitions of Terms and Symbols

a. Reference Frames.

(1) Figures 9, 10, and 11 illustrate typical reference frames for obtaining parameters of the out-of-level problem. These reference frames will be considered in two groups. The first group comprises the level and non-level reference frames of the weapon proper which is that part of the overall equipment that traverses and elevates to the direction of fire, i.e., the weapon tube and turret. This part of the overall weapon will be referred to throughout the text as the "weapon." The second group of reference frames applies to the undercarriage which henceforth will be referred to as the "weapon mount" or "mount." In the typical on-carriage fire control system shown in Figures 9, 10, and 11, the "weapon" consists of the gun and turret while the "weapon mount" is the tank hull.

(2) The three orthogonal axes of *Fa* represent the level reference frame of the weapon (Figure 9). Cant and pitch angles whose axes are horizontal are referenced to the *Fa* frame. The *Fa* frame also contains the vertical axis about which all horizontal azimuth angles are measured. The three orthogonal axes of *Fg* represent the out-of-level axes of the weapon. This frame constitutes the trunnion axis, the traverse axis of the turret, and the weapon bore axis. The traverse axis is perpendicular to the deck plane while the trunnion axis and the weapon bore axis (at zero elevation) lie in the deck plane. In a similar manner, reference frames *Fr* and *Fm* form the basis for establishing the out-of-level condition of the weapon mount. The three axes of *Fr* represent the fixed, level reference frame of the mount while the axes of *Fm* represent the mechanical reference frame of the non-level mount, or hull. The axes of *Fr* and *Fm* are aligned with the longitudinal and cross axes of the mount. An additional reference frame, *Fs*, containing the axes of the aiming device will be employed, but in the on-carriage system shown in Figures 9, 10, and 11, it coincides with *Fg* except for a small linear displacement (parallax).

(3) The angle symbols used in the mathematical expressions in this handbook are based, wherever possible, on the identifying lower case letter of their associated reference frame symbols. Cant angle, for example, is designated *Cm* when associated with the mount reference frame (*Fm*) and designated *Cg* when associated with the weapon reference frame (*Fg*) containing the weapon trunnions.

(4) In Figure 9 the weapon bore axis is aligned with the weapon mount longitudinal axis, so that the trunnion cant angle (*Cg*) and the weapon mount cant angle (*Cm*) are equal as are the weapon bore pitch angle (*La*) and the weapon mount pitch angle (*Lr*). In Figure 10 as the weapon bore axis rotates in azimuth (angle *Ad*) *Fa* and *Fg* rotate with it and the trunnion cant angle (*Cg*) and the weapon bore pitch angle (*La*) begin to differ from the weapon mount cant and pitch angles (*Cr* and *Lr*), respectively. When a relative azimuth angle of 90 degrees is reached, the trunnion cant angle (*Cg*) and the weapon mount pitch angle (*Lr*) are equal as are the weapon bore pitch angle (*La*) and the weapon mount cant angle (*Cm*). The amount of elevation above the deck plane is equal to the angle (*Eg*) between the weapon bore axis at zero elevation and the weapon bore axis elevated to firing position. (See Figure 11.) The true elevation above a horizontal plane is equal to the angle (*Eo*, quadrant elevation) between the cant axis at the given relative azimuth (*Ad*) and the weapon bore axis.

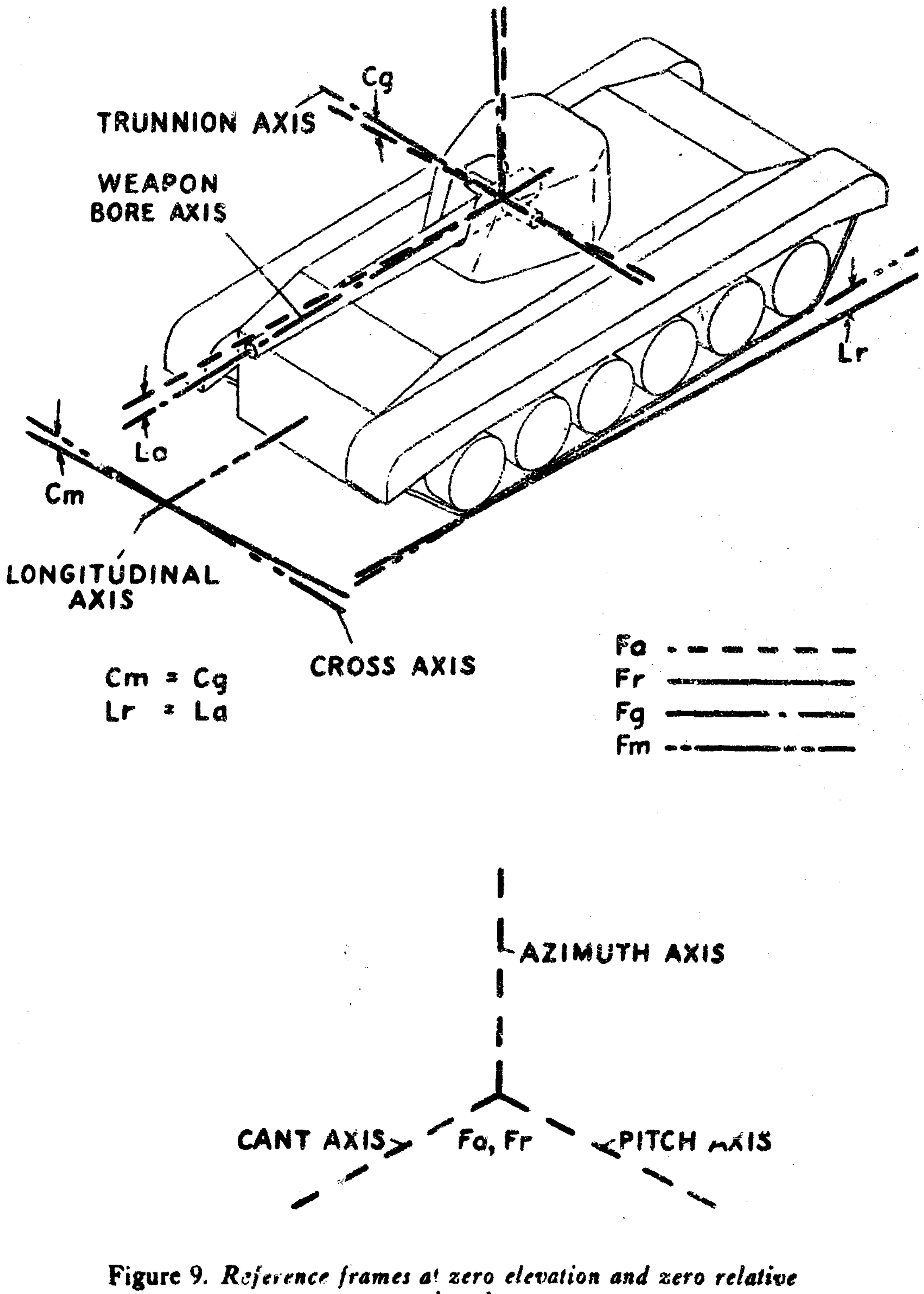


Figure 9. *Reference frames at zero elevation and zero relative azimuth*

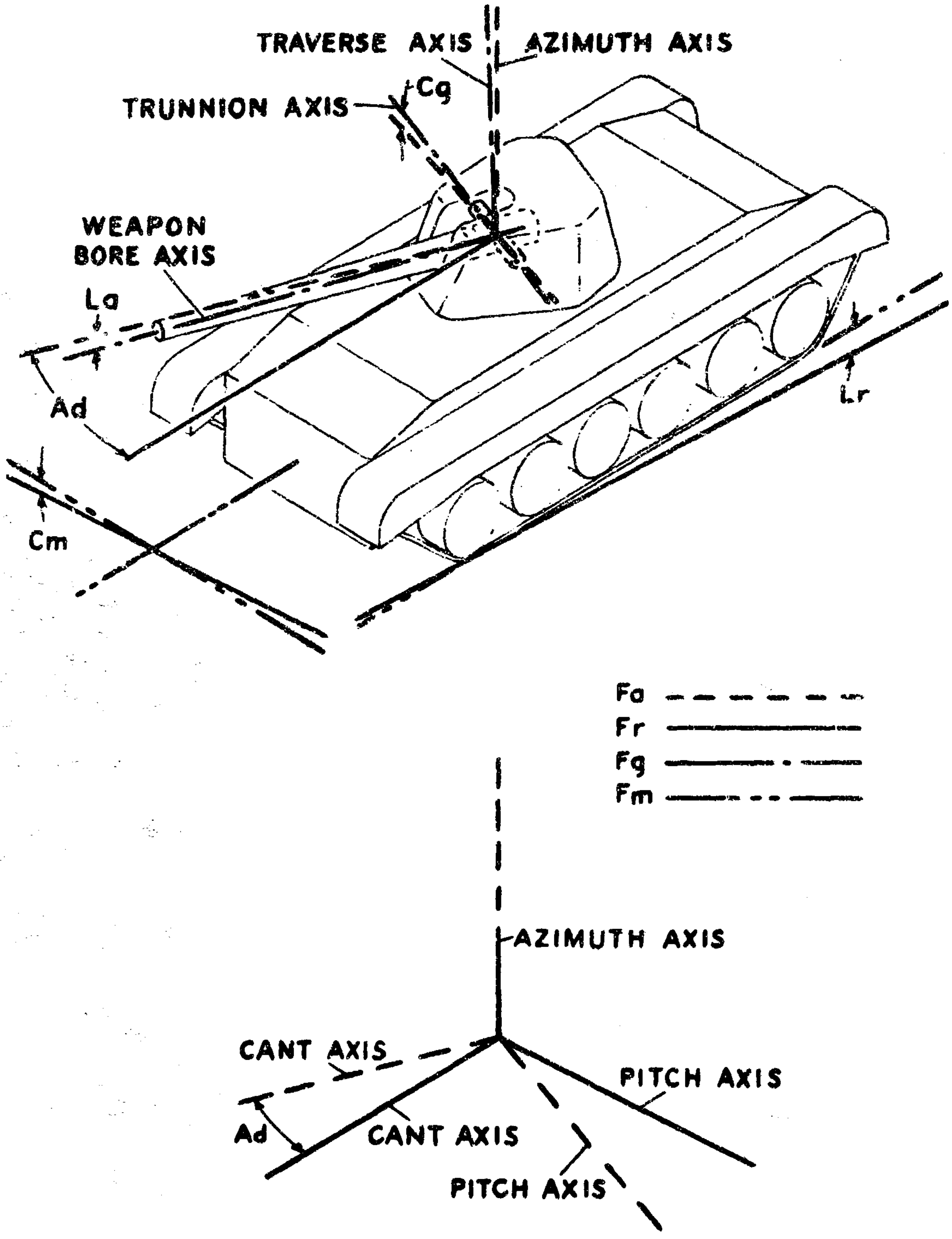


Figure 10. *Weapon reference frame at relative azimuth angle Ad—elevation zero*

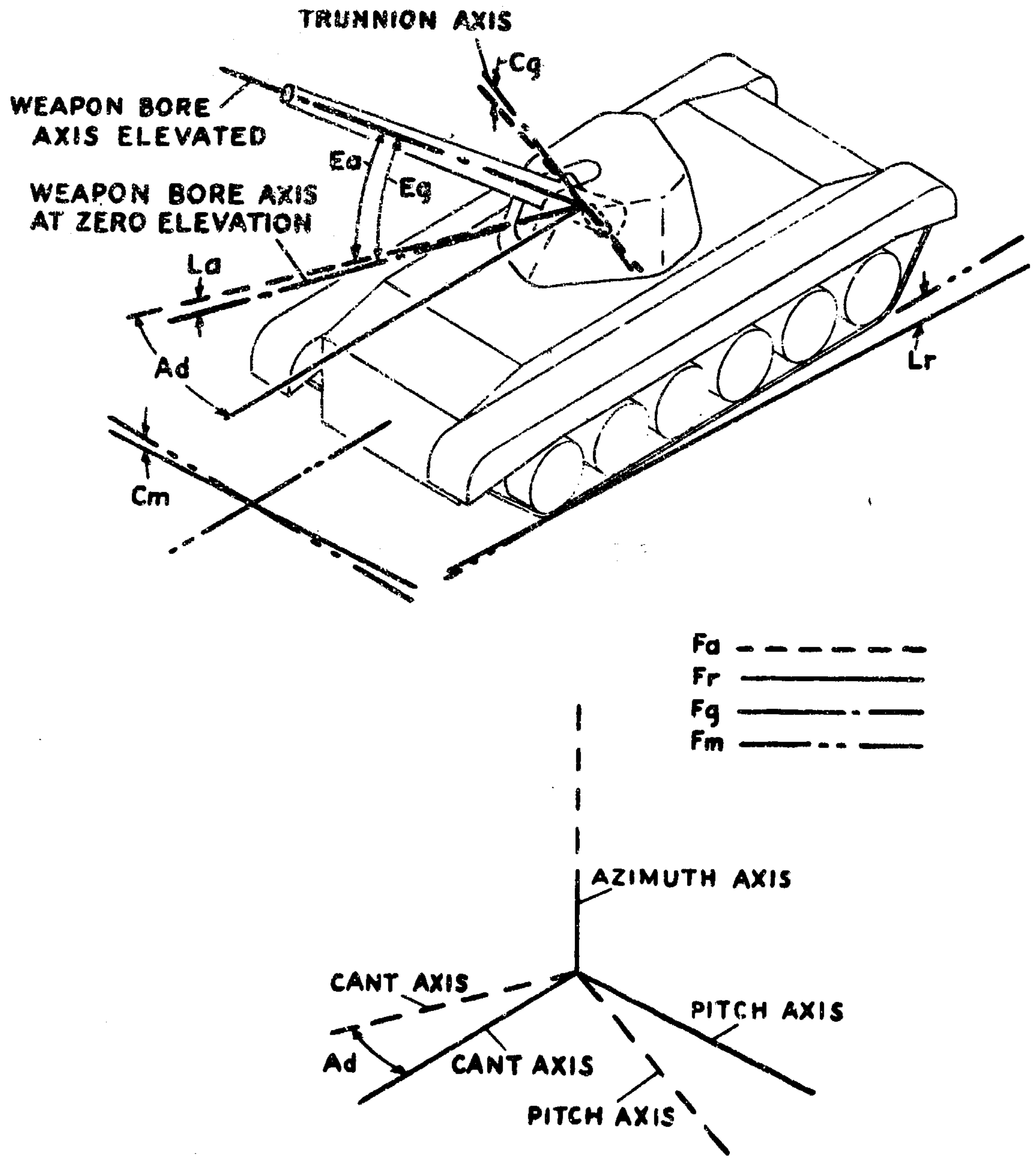


Figure 11. *Weapon reference frames traversed and elevated*

b. Symbols. The various angles defined in the following text are identified in Figures 9 through 19.

(1) Azimuth (A). The horizontal angle between north and the direction to the target. Other horizontal azimuth angles used in this text are:

(a) Ag—True azimuth of weapon, measured between north and vertical plane through the weapon bore axis (Figure 13).

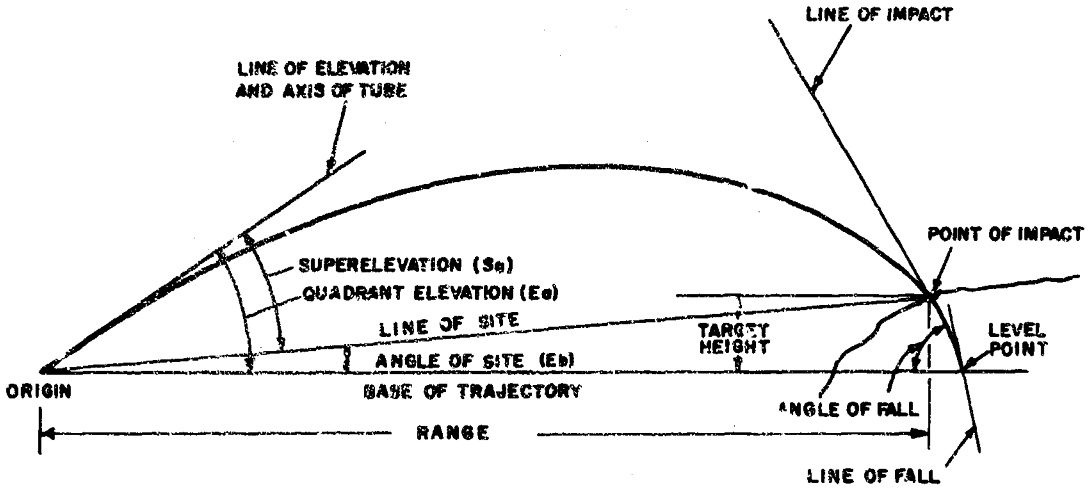


Figure 12. *Trajectory terms*

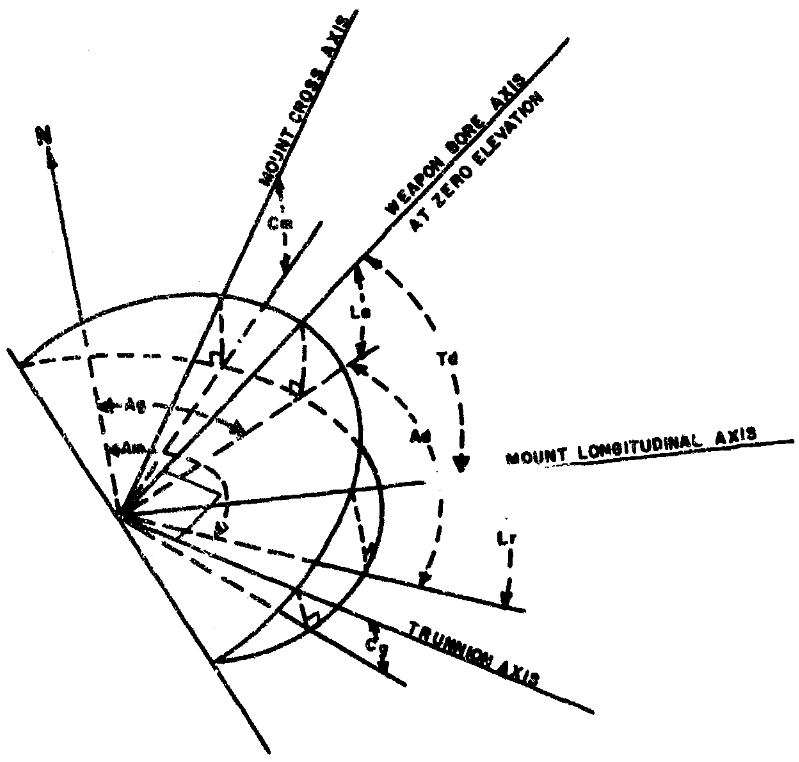


Figure 13. *Relationship between mount tilt and weapon tilt*

(b) *Am*—Azimuth angle between north and a vertical plane through the longitudinal axis of the weapon mount (Figure 13).

(c) *Ad*—Relative azimuth or difference between true azimuth of weapon (*Ag*) and the true azimuth of the longitudinal axis of the mount (*Am*) (Figure 13).

(d) *Ao*—Azimuth of vertical plane through weapon bore axis relative to deck plane tilt axis (Figure 14).

(e) *At*—Azimuth of vertical plane through weapon bore axis relative to azimuth direction of deck plane tilt (Figure 14).

(f) *Agd*—Difference in azimuth between the weapon elevating plane and the horizontal projection of the weapon bore axis (Figure 17). The angle *Agd* is the correction required to return the weapon to its original azimuth after elevating with the trunnions canted.

(g) *Asd*—Difference in azimuth between the aiming device elevating plane and the horizontal projection of the aiming device axis. (Figure 19).

(2) Gun Elevation (*Eg*). Angle of the weapon bore with respect to the deck plane. The angle is measured in a plane (elevating plane) perpendicular to the deck plane (Figures 14 and 17).

(3) Quadrant Elevation (*Es*). The quadrant elevation is the vertical angle at the origin formed by the line of elevation and the base of the trajectory. It is the algebraic sum of the angle of elevation (superelevation) and the angle of site, (Figures 12 and 17).

(4) Line of Elevation. The line of elevation is the axis of the tube prolonged when the piece is laid (Figure 12).

(5) Base of Trajectory. The base of the trajectory is the straight line from the origin to the level point (Figure 12).

(6) Level Point. The level point is the point on the descending branch of the trajectory which is at the same altitude as the origin (Figure 12).

(7) Superelevation (*Se*). This "angle of elevation" is the vertical angle at the origin between

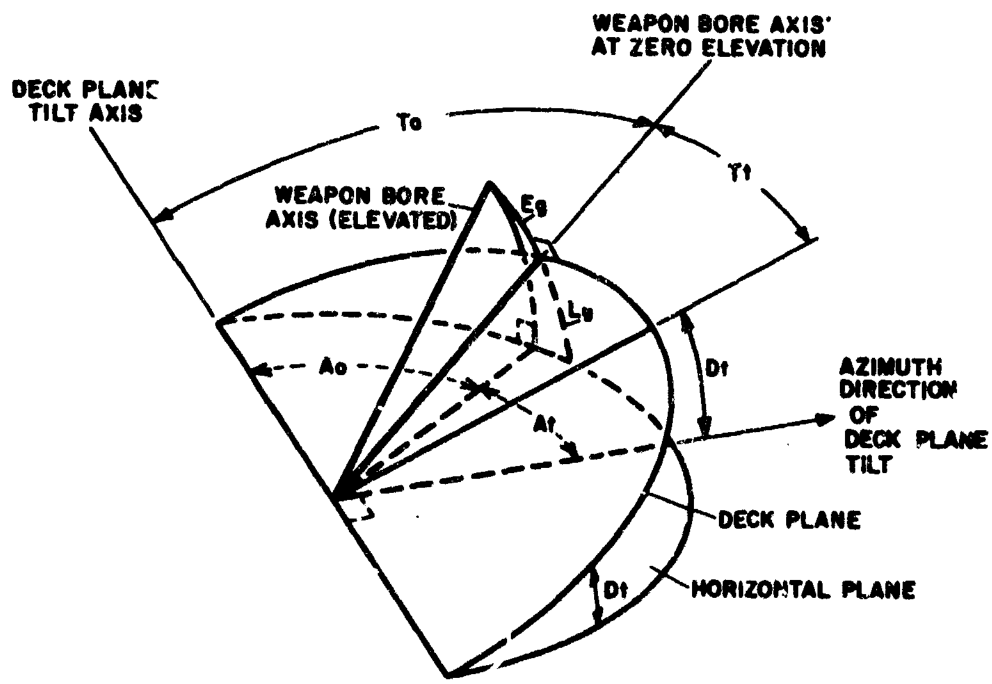


Figure 14. *Deck tilt magnitude and direction relative to weapon position*

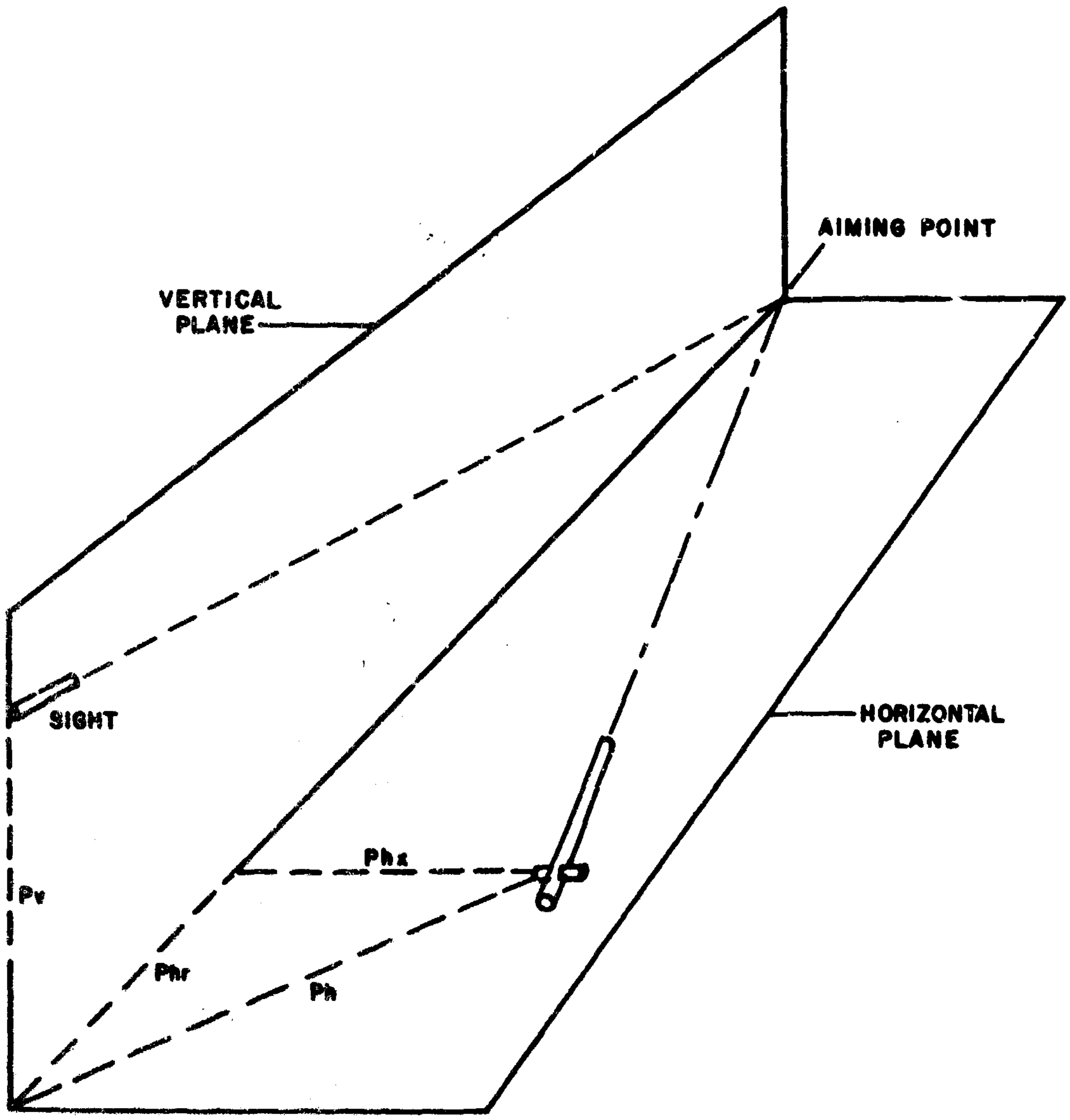


Figure 15. *Parallax in a specific-range boresighted system*

the line of site and the line of elevation (Figure 12). Transformed to canted plane of weapon elevation, angle becomes *Seg*.

(8) Angle of Site (*E* or *Eb*). The angle of site is the vertical angle between the line of site and the horizontal plane. Measured at the position of the aiming device, it is designated *E*. Measured at the position of the weapon, between the line to the target and the base of the trajectory, the angle is designated *Eb* (Figures 12 and 25).

(9) Aiming Device Elevation (*Es*). The angle between the line of site and the aiming device deck plane, measured in a plane perpendicular to the deck plane (Figure 19).

(10) Trunnion Cant (*Cg* or *Cs*). Tilt of the weapon (or aiming device) trunnions with respect to a horizontal reference plane. The angle is measured in a vertical plane (Figure 13).

(11) Weapon Mount Cant (*Cm*). Tilt of the weapon mount cross axis with respect to a

ho.'--ntal reference plane. The angle is measured in a vertical plane (Figure 13).

(12) Weapon Bore Vertical Pitch (La). Tilt of the deck plane in the direction of the weapon bore axis, measured in the vertical plane between the horizontal plane and the intersection of the deck and elevating planes (Figures 13 and 17).

(13) Weapon Bore Pitch (Lg). Similar to La but measured in the canted weapon elevating plane. When Cg = zero, $Lg = La$ (Figures 14 and 17).

(14) Weapon Mount Vertical Pitch (Lr). Tilt of the weapon mount in the fore-and-aft direction with respect to a horizontal reference plane. The angle is measured in a vertical plane (Figure 13).

(15) Weapon Mount Pitch (Lm). Similar to Lr but measured in a plane normal to the deck plane.

(16) Aiming Device Pitch (Ls). Tilt of the aiming device deck plane in the direction of the line of site as measured in the elevating plane containing the line of site between the horizontal and deck planes.

(17) Deck Plane Tilt (Dt). The angle between the deck plane and the horizontal reference plane, measured at right angles to the horizontal deck plane tilt axis (Figure 14).

(18) Traverse (T). Rotation of the weapon about the traverse axis, measured in the deck plane. Specific traverse angles employed in this text are:

(a) Td—Traverse angle between weapon elevating plane and longitudinal axis of weapon mount (Figure 13).

(b) To—Traverse angle between weapon elevating plane and deck plane tilt axis (Figure 14).

(c) Tt—Traverse angle between weapon elevating plane and azimuth direction of deck plane tilt (Figure 14).

(d) Tgd—Difference in traverse between the weapon elevating plane and the horizontal projection of the weapon bore axis. Angle Tgd is the traverse correction required to return the weapon to its original azimuth after elevating with the trunnions canted (Figure 17).

(e) Tsd—Difference in traverse between the aiming device elevating plane and the horizontal projection of the aiming device axis (Figure 19).

(19) Parallax (P). Apparent differences in the position of a target viewed from a weapon position and a directing or sighting point (Figures 15 and 16).

(20) Parallax Base. The displacement between the weapon position and directing or sighting point as measured in a horizontal (Ph), vertical (Pv), or range (Pr) direction (Figures 15 and 16).

10. Discussion of Effects Caused by Shift of Reference Frame

a. General. Before going into the mathematical treatment of compensation problems, some of the effects of reference frame displacement will be discussed briefly. This discussion introduces the various compensation problems for which equations are presented in subsequent paragraphs.

b. Off-Carriage Fire Control.

(1) Trunnion Cant. If the data furnished by an off-carriage sight is not corrected for trunnion cant, the range obtained will be less than the desired range and an azimuth deflection will be set into the weapon. The component of deflection that is introduced into the angle of elevation by trunnion tilt is small enough to be neglected in practical cases unless extreme accuracy is required.

(2) Weapon Bore Pitch. A range error will result if the elevation data furnished by the off-carriage sight is not corrected for weapon bore pitch.

(3) Parallax. Parallax can produce errors in range, azimuth and elevation. If a sight is located behind or in front of a weapon, errors in range result. If a sight is located to one side of a weapon, horizontal errors are produced. Vertical errors (resulting in range errors) are introduced when a sight in mounted above or below a weapon.

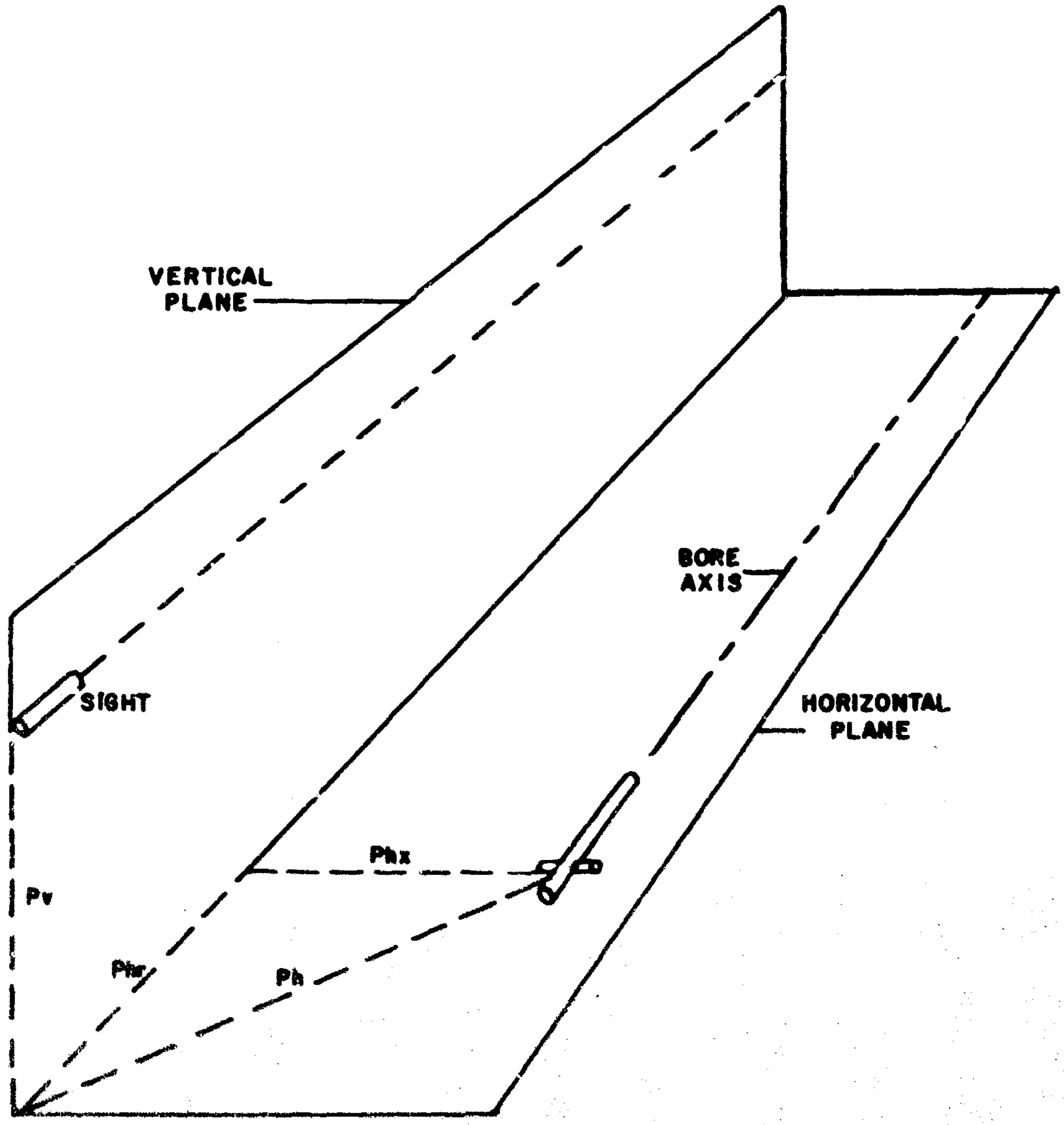


Figure 16. *Parallax in an infinity-boresighted system*

c. On-Carriage Fire Control.

(1) Trunnion Cant. Trunnion cant operates to reduce range and to cause lateral deviation toward the side of the lower trunnion. The component of elevation that affects the azimuth setting may cause very serious errors in azimuth. The component of elevation that is lost due to trunnion cant may, under exceptional circumstances, cause noticeable errors in range, but ordinarily these errors are small.

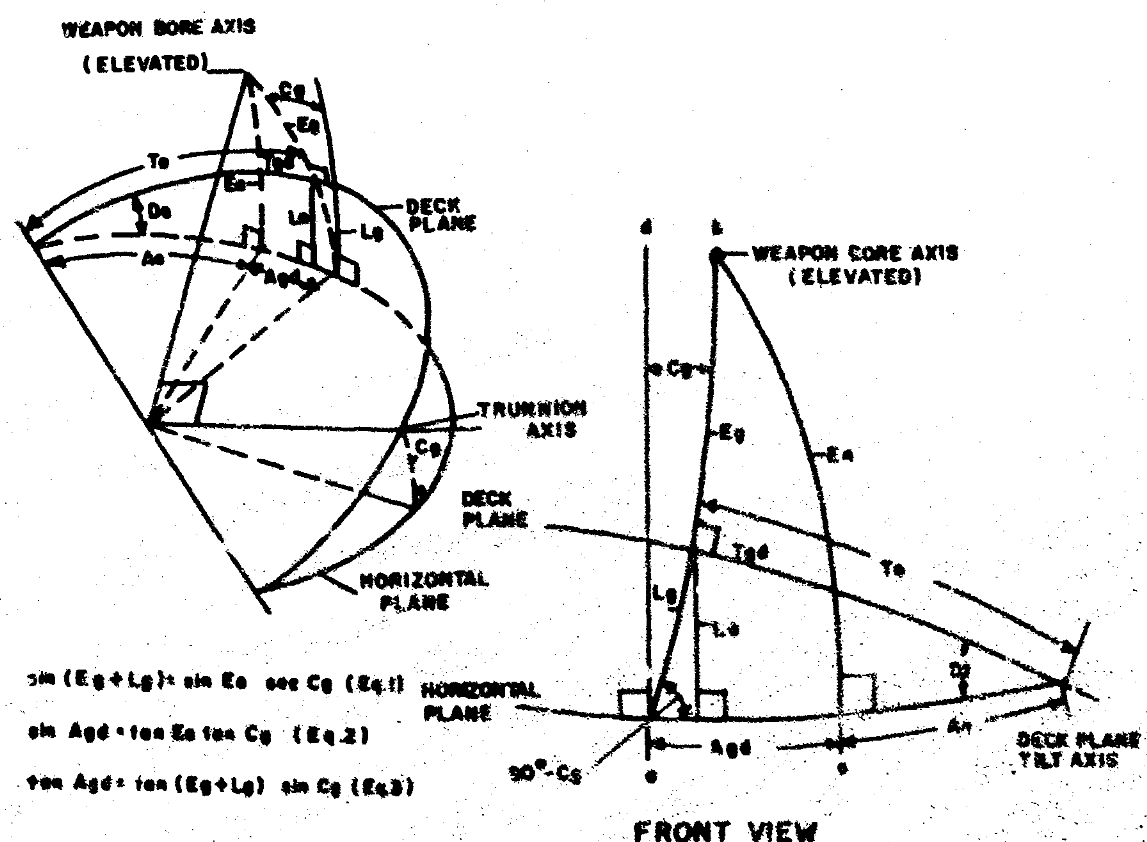
(2) Weapon Bore Pitch. Errors in elevation setting will result if weapon bore pitch is not taken into consideration.

(3) Parallax. Parallax produces slight errors in range, azimuth, and elevation which may be partially compensated for by adjustment during boresighting.

11. Mathematical Treatment of Compensation Problems

a. The usual approach to an engineering problem often requires that the mathematics of the problem be resolved in the initial steps of design. A mathematical analysis facilitates the understanding of compensation problems. Such a mathematical approach is given in the following paragraphs. An attempt has been made to give several approaches for many different situations requiring compensation. However, it is possible that situations will arise that have not been treated in this book. In this case, the approach to the compensation problems given here will provide guidelines for procedures in solving the problem presented.

b. Before going into a presentation of the equations for situations requiring compensation, it is well to give the reader some background in mathematical approach. Mathematical expressions can be obtained easily from a geometric configuration representing an out-of-level compensation problem using plane trigonometric relationships. As simple as this approach might be, it has the disadvantage of producing expressions that require tedious and time-consuming manipulations. Another method that can be used to obtain mathematical expressions employs spherical trigonometry relationships. This approach usually results in equations obtained by more direct methods. However, since spherical trigonometry is not always taught in the mathematical sequence of an engineering curiculum, rules for obtaining expressions, derivations, and identities are included in Appendixes A and B. Readers not familiar with the subject may refer to a mathematics text including spherical trigonometry if additional information is needed.



$$\sin (E_g + L_g) = \sin E_o \sec C_g \quad \text{(Eq. 1)}$$

$$\sin A_{gd} = \tan E_o \tan C_g \quad \text{(Eq. 2)}$$

$$\tan A_{gd} = \tan (E_g + L_g) \sin C_g \quad \text{(Eq. 3)}$$

Figure 17. *Elevation of bore axis w[illegible] trunnions canted*

c. One of the basic difficulties for persons new to spherical trigonometry, as used in compensation problems, is to establish the correct angles for the spherical triangle. For example, Figure 17 illustrates a weapon elevated with trunnions canted. Assume that the problem is to find the error in azimuth caused by trunnion cant. Further, let us make the stipulation that we wish to express the error in terms of gun elevation ($Eg + Lg$) and cant (Cg). At first glance, one would expect the angle formed by Ea and Eg meeting at the weapon bore (front view, $<abc$) to be equal to Cg. This reasoning is incorrect since the angle between Ea and Eg is on a sphere and is displaced spherically from the cant angle; hence, the rule of plane geometry on which the assumption is made is not valid. However, by constructing a perpendicular (ad) to the horizontal plane at point a, it can be seen that the angle formed ($<bad$) is Cg. Then $<bac$ in spherical triangle abc is equal to $90° - Cg$. Once the angle is determined correctly, Napier's rules can be used to write an expression. (See Appendix B.) To illustrate:

from Napier's rules:

$$\sin\left[\text{Comp. } (90 - Cg)\right] = \tan Agd \tan\left[\text{Comp. } (Eg + Lg)\right]$$

solving:

$$\sin\left[90 - (90 - Cg)\right] = \tan Agd \tan\left[90 - (Eg + Lg)\right]$$

$$\sin Cg = \tan Agd \cot (Eg + Lg)$$

$$\sin Cg = \tan Agd \frac{1}{\tan (Eg + Lg)}$$

$$\tan Agd = \sin Cg \tan (Eg + Lg)$$

(See Eq. 3)

Similarly:

$$\sin Lg = \tan To \tan Cg \quad \text{(See Eq. 18)}$$

$$\sin Lg = \frac{\sin La}{\cos Cg} = \sin La \sec Cg$$

$$\sin La = \sin To \sin Dt$$

$$\text{and } \sin To = \cot Dt \tan Lg$$

d. The following general procedures have been found helpful for solving out-of-level compensation problems:

(1) Draw a schematic or pictorial diagram of the situation. Make certain the figure is correct. (Many hours can be expended in obtaining a solution from a figure that does not illustrate the actual problem at hand.)

(2) Use systemitized notational symbols to avoid confusion, e.g., C for cant, Cg for gun trunnion cant, Cm for cant of mount, etc.

(3) Choose a triangle in the figure that can give an expression which includes the quantity you wish to correct.

(4) Be sure that all sides of the chosen triangle are portions of great circles.

(5) Make certain the quantities in the final expression are expressed in terms that can be measured in a practical system. (Often an expression will contain quantities that simplify mathematical manipulations but make instrumentation difficult or impractical.)

12. Cant Corrections

The following paragraphs are a presentation of equations and mathematical considerations pertaining to cant correction.

13. Off-Carriage Fire Control Systems

In an off-carriage fire control system, the sight or data-computing equipment is not mechanically coupled to the weapon, and its reference frame is different from the mechanical reference frame of the weapon. Consequently, gun laying data derived from an off-carriage aiming device will produce erroneous aiming settings if applied to an out-of-level weapon without being converted into the reference frame of the weapon. Also, if the off-carriage source is out-of-level, additional errors will be introduced.

a. Weapon Out-of-Level, Aiming Device Level.

(1) Elevation Error. If a weapon is to be elevated vertically by an amount Ea, as determined by an off-carriage sight, the actual elevation of the weapon (Eg) about canted trunnions may be computed from the following equation (Figure 17).

$$\sin (Eg + Lg) = \sin Ea \sec Cg \quad \text{(Eq. 1)}$$

(2) Azimuth Error. The amount of azimuth change, as the gun bore is elevated on a canted trunnion, from zero elevation to a new elevation may be computed from either of the following equations (Figure 17).

$$\sin Agd = \tan Ea \tan Cg \quad \text{(Eq. 2)}$$

$$\tan Agd = \tan (Eg + Lg) \sin Cg \quad \text{(Eq. 3)}$$

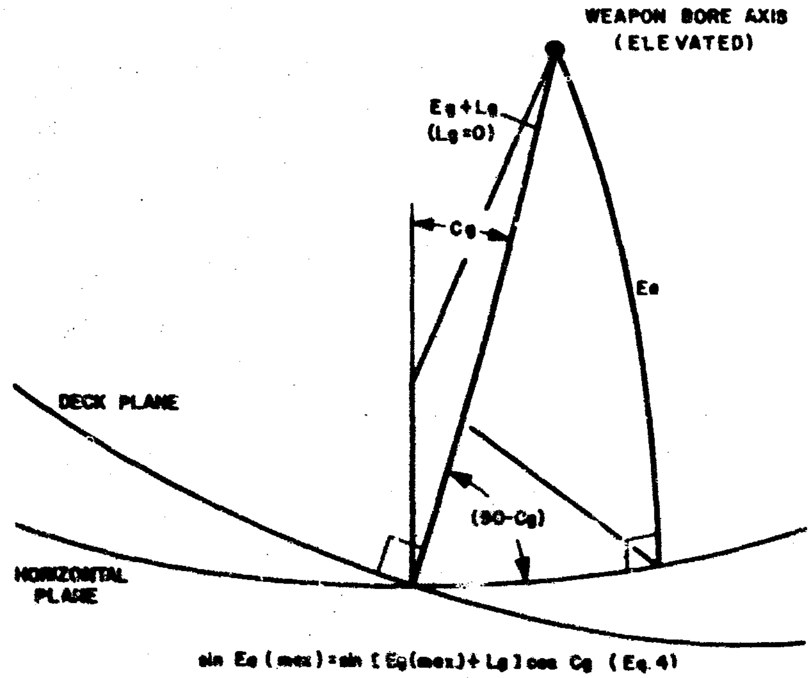


Figure 18. *Effect of cant on maximum elevation*

(3) Effect of Cant on Maximum Elevation. Because the secant function for any cant angle other than zero is greater than 1, Equation 1 shows that greater elevation ($E_g + L_g$) about a canted trunnion is needed to provide a given vertical elevation (E_e) than when cant is not present. The pitch angle (L_g), shown positive in Figure 17, may be either positive or negative. Measured in the same plane as gun elevation, L_g affects maximum gun elevation directly. Positive pitch increases the maximum attainable E_g and tends to cancel the effects of cant while negative pitch further limits the maximum E_g. The effect of cant is therefore better illustrated when pitch is shown at zero as in Figure 18. Equation 4 expresses the maximum vertical elevation attainable, E_e (max.), as a function of maximum gun elevation, E_g (max.), and cant, C_g.

$$\sin E_e \text{ (max.)} = \sin(E_g \text{ (max.)} + L_g) \cos C_g \quad \text{(Eq. 4)}$$

b. Weapon Level, Aiming Device Out-of-Level.

(1) Elevation Error Caused by Cant of Aiming Device. If the aiming device is out-of-level, measured elevation angles will be incorrect for a leveled weapon. The canted sight elevation angle (E_s) may be converted to the vertical angle of site (E) using the following equation (Figure 19).

$$\sin E = \sin (E_s + L_s) \cos C_s \quad \text{(Eq. 5)}$$

(2) Azimuth Error Caused by Cant of Aiming Device. If the aiming device is out-of-level, a false azimuth angle will be applied to the leveled weapon. The error in azimuth may be computed from the following equation (Figure 19).

$$\tan A_{sd} = \tan (E_s + L_s) \sin C_s \quad \text{(Eq. 6)}$$

c. Weapon Out-of-Level, Aiming Device Out-of-Level.

(1) Elevation Error Caused by Cant in Weapon and Aiming Device. The conversion of aiming device elevation to weapon elevation, when both units are canted, can be accomplished in three steps:

(a) Convert aiming device elevation in the canted plane (E_s) to vertical elevation (E) by means of Equation 5.

(b) Convert vertical elevation (E) to quadrant elevation (E_e) by adding the necessary firing corrections such as superelevation and lead angle.

(c) Convert quadrant elevation (E_e) to weapon elevation (E_g) using Equation 1.

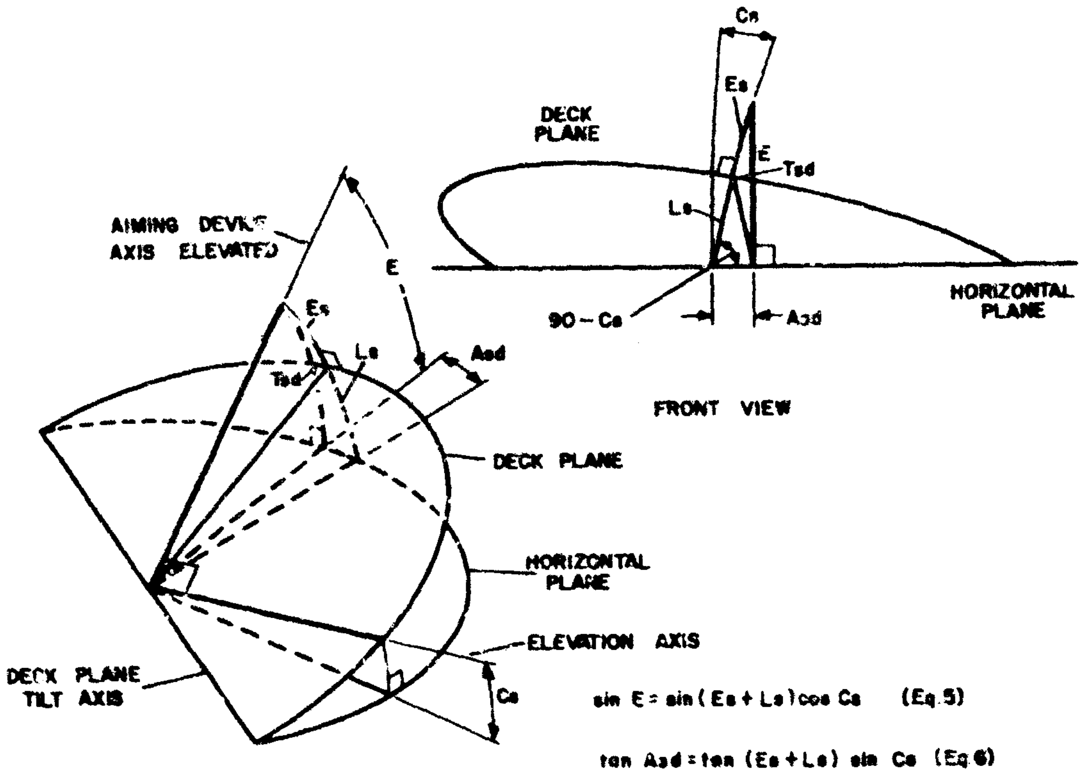


Figure 19. *Aiming device out-of-level*

(2) Azimuth Error Caused by Cant in Weapon and Aiming Device. The azimuth error (A_{gd}) may be computed by adding the error caused by the canted trunnion of a weapon (Equation 3) to the azimuth error contributed by a canted aiming device (Equation 6). The result is:

$$A_{gd} = \text{arc tan}\left[\tan (E_g+L_g)\sin C_g\right] + \text{arc tan}\left[\tan (E_s+L_s) \sin C_s\right] \quad \text{(Eq. 7)}$$

14. On-Carriage Fire Control Systems

In an on-carriage fire control system the weapon follows the aiming device in elevation and traverse as it seeks to establish a line of site (paragraph 4.a.(1)). However, independent elevation of the weapon bore for insertion of superelevation (S_e) requires compensation in both elevation and azimuth, if the system trunnions are canted.

a. Elevation Angle Measured in a Vertical Plane.

(1) Elevation Error. The elevation error caused by cant can be compensated by transforming superelevation to the gun elevating plane (Figure 20). Correcting the resultant azimuth error produces the geometry shown in the figure. The elevation correction can be computed from the vertical elevation angles and cant angles as follows:

$$S_{eg} = (E_g+L_g)-(E_s+L_s)$$

$$S_{eg} = \text{arc sin } (\sin E_a \sec C_g) - \text{arc sin } (\sin E_b \sec C_s) \quad \text{(Eq. 8)}$$

(2) Azimuth Error. Using the same triangles employed in computing the elevation correction (Figure 20), the azimuth error may be compensated through the means of a similar equation:

$$A_{gd} - A_{sd} = \text{arc sin } (\tan E_a \tan C_g) - \text{arc sin } (\tan E_b \tan C_s) \quad \text{(Eq. 9)}$$

The compensation also may be computed in terms of traverse angles and the angle of deck plane tilt (D_t):

$$T_{gd}-T_{sd} = \text{arc sin } (\cot D_t \tan L_g) - \text{arc sin } (\cot D_t \tan L_s) \quad \text{(Eq. 10)}$$

b. Elevation Angle Measured in Plane Perpendicular to Trunnion.

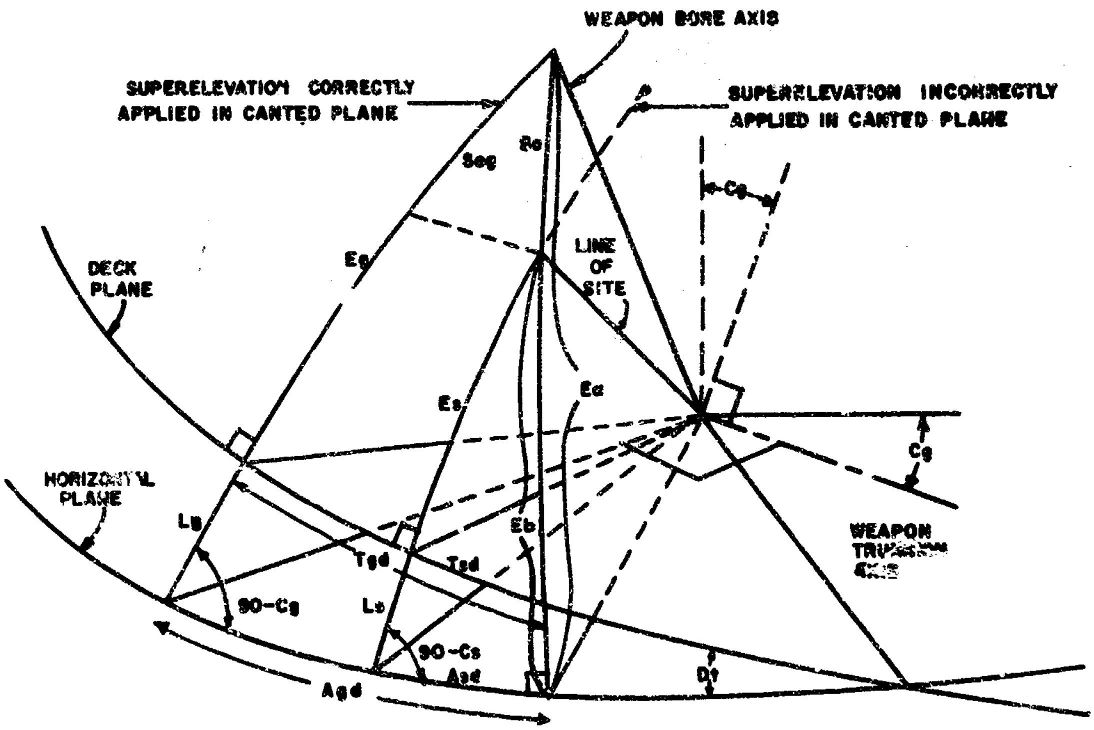


Figure 20. *On-carriage fire control compensation problem*

(1) Elevation Error. Compensation for the elevation error in terms of angles in the canted elevating planes is (Figure 20):

$Se = Ea - Eb$

$Se = \text{arc sin}\left[\sin(Eg+Lg)\cos Cg\right] - \text{arc sin}\left[\sin(Es+Ls)\cos Cs\right]$ (Eq. 11)

(2) Azimuth Error. The azimuth compensation based on canted elevation angles is:

$Agd - Asd = \text{arc tan}\left[\tan(Eg+Lg)\sin Cg\right] - \text{arc tan}\left[\tan(Es+Ls)\sin Cs\right]$ (Eq. 12)

c. Effect of Cant on Maximum Elevation. The effect of cant on maximum attainable weapon elevation is the same as for the conditions covered in paragraph 13.a.(3).

d. Equations for Cant Compensation by Reticle Rotation. If a direct fire sight reticle is rotated about the boresight mark from a canted position to a vertical position so that superelevation (Se) may be applied vertically, the errors expressed by Equations 13 and 14 are eliminated. (See Figures 7 and 21 and paragraph 5.a.(2).)

$x = \text{arc sin}(\sin Se \sin C)$ (Eq. 13)

$v = Se - \text{arc tan}(\sin C \tan Se)$ (Eq. 14)

where:

x is the azimuth error,

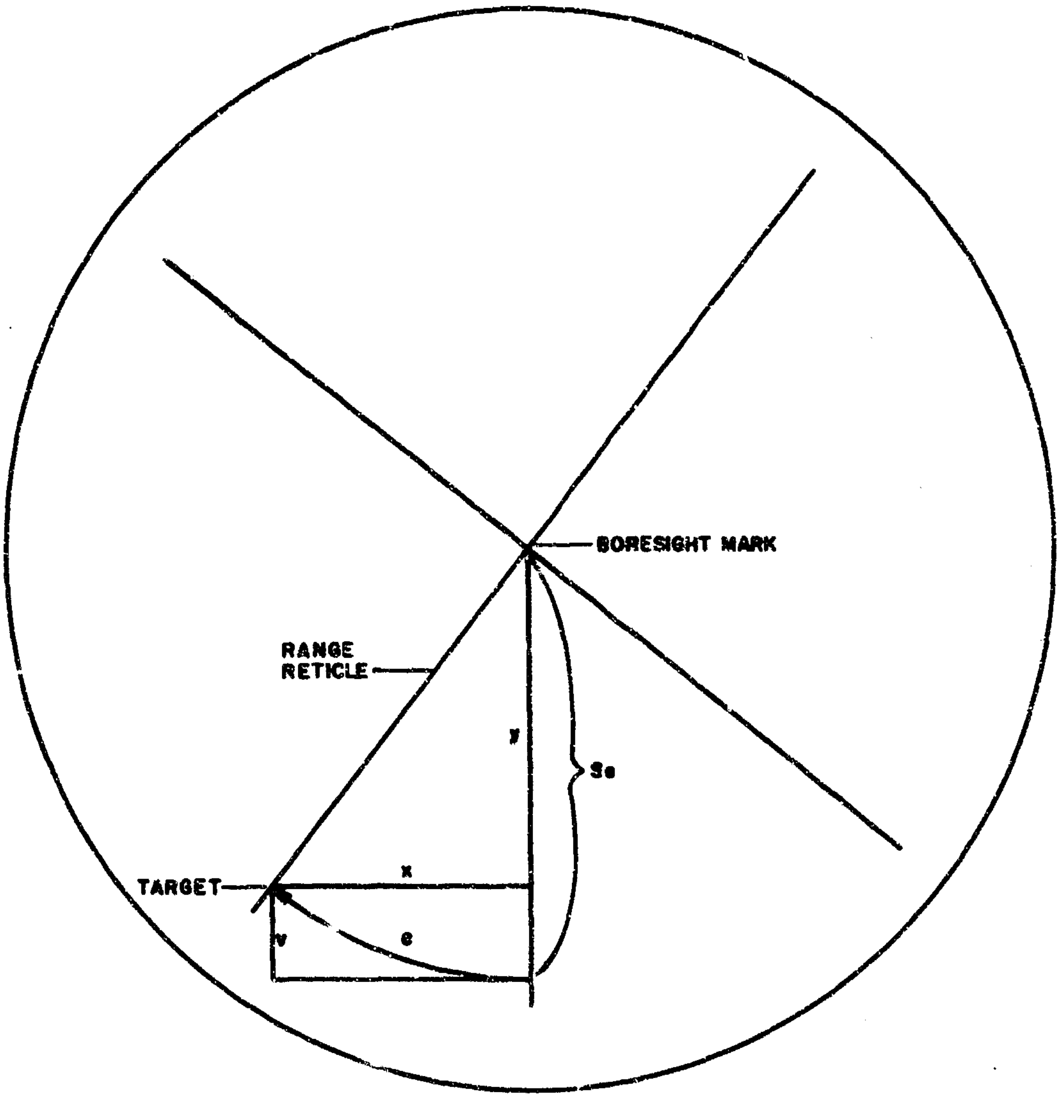
v is the elevation error,

C is a cant angle measured about the boresight axis.

15. Relationship Between Mount Tilt and Weapon Tilt

Since the out-of-level conditions of a weapon mount do not vary with azimuth as do trunnion cant and gun bore pitch, it is often simpler to measure gun mount cant and pitch. In Figure 22 trunnion cant (Cg) and gun bore pitch (Lg) may then be computed from gun mount cant (Cm), gun mount pitch (Lm), and relative traverse (Td), by means of the following equations: (See Appendix C for derivation of Equations 15 and 16.)

$\cos Lg(-\cos Td \sin Lm \cos Cm + \sin Td \sin Cm) + \sin Lg \cos Lm \cos Cm = 0$ (Eq.15)



AZIMUTH ERROR = x = arc sin (sin Se sinC) (Eq. 13)

ELEVATION ERROR = v = Se − arc tan (sin C tan Se) (Eq. 14)

Figure 21. *Errors resulting from canting of sight reticle about the boresight mark*

$$\cos Cg\,(-\sin Td \sin Lm \cos Cm - \cos Td \sin Cm) + \sin Cg\,[-\sin Lg\,(-\cos Td \sin Lm \cos Cm + \sin Td \sin Cm) + \cos Lg \cos Lm \cos Cm] = 0 \quad \text{(Eq. 16)}$$

16. Inter-Relationship of Cant and Pitch

Equations 17, 18, and 19 show the inter-relationship of pitch and cant angles, with one other angle used as a basis for comparison. The angles are illustrated in Figure 17.

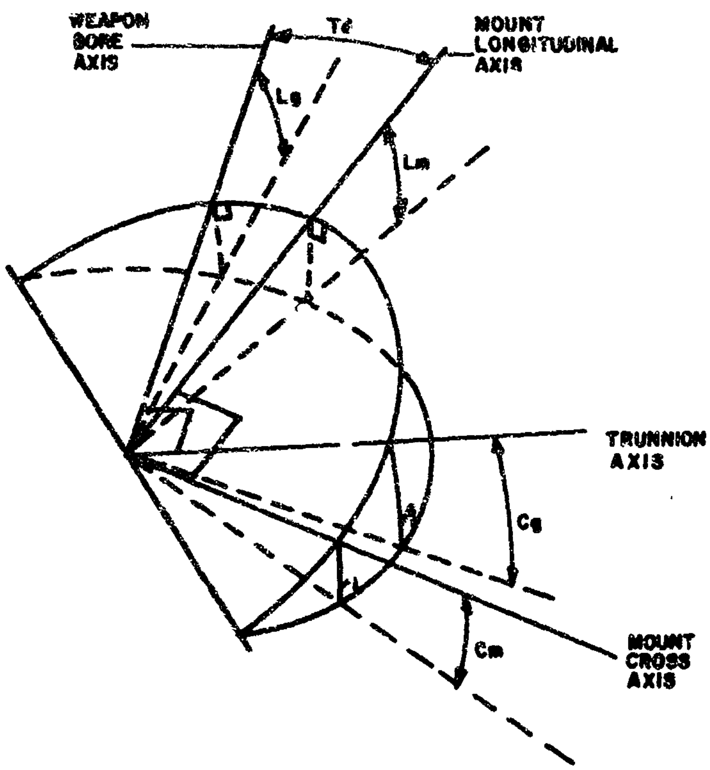


Figure 22. ***Relationship between weapon mount cant and pitch and weapon bore cant and pitch***

$$\cos L_g = \sec C_g \cos D_t \qquad \text{(Eq. 17)}$$

$$\sin L_g = \tan C_g \tan T_o \qquad \text{(Eq. 18)}$$

$$\tan L_g = \sin C_g \tan (A_o + A_{gd}) \qquad \text{(Eq. 19)}$$

17. Combined Correction for Cant and Pitch

a. It is possible to combine the effects of cant and pitch so that errors are measured or corrections made in terms of an azimuth angle (A_t) and tilt angle (D_t). This approach has been tested in a prototype antiaircraft weapon as described further in paragraph 29. (See Figure 23.) The equation given here are derived in Appendix C.

b. The azimuth of the weapon relative to the direction of tilt (A_t) is given by:

$$\tan A_t = \frac{\sin T_t}{\cos T_t \cos D_t + \tan E_g \sin D_t}; \qquad \text{(Eq. 20)}$$

and the quadrant elevation of the gun is:

$$\sin E_a = \sin E_g \cos D_t - \cos E_g \cos T_t \sin D_t \qquad \text{(Eq. 21)}$$

18. Parallax Corrections

a. Off-carriage fire control systems such as might be employed in the typical antiaircraft battery shown in Figure 24 have relatively large displacements between the directing radar and the individual weapons. Separate parallax corrections can be applied by the director to the laying data for each weapon so that the fire from all weapons in a battery will converge on a target. Figure 24 shows how the displacements can be measured during the emplacement process for use in parallax computations. The following paragraphs present equations for computing parallax corrections both with polar coordinates and rectangular coordinates. It will also be shown how the equations reduce to simpler forms when the problem concerns only ground target systems, as in field artillery. The general principles illustrated by these cases can be applied to other parallax situations requiring convergence of lines.

b. The equations based on polar coordinates express corrections for vertical and horizontal displacements in terms of angles that can be combined

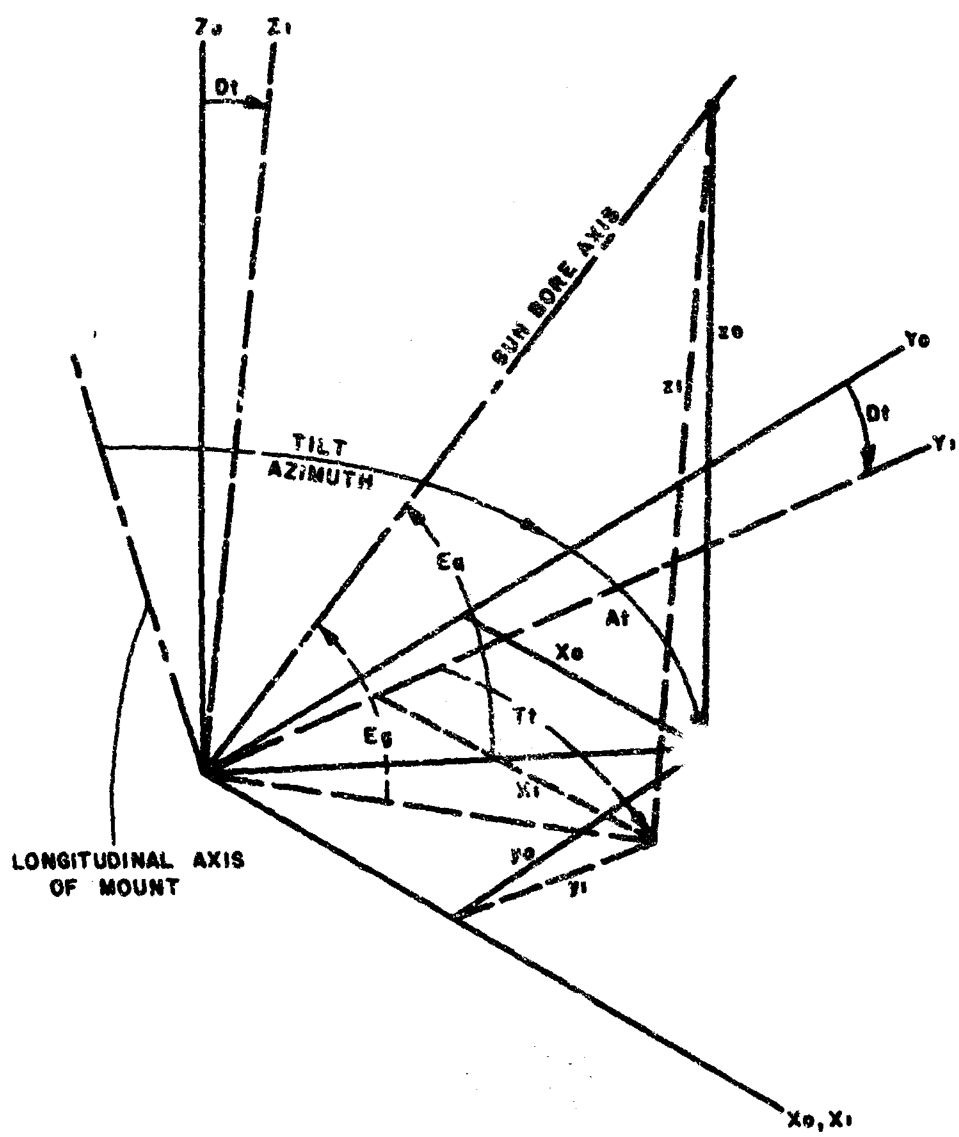


Figure 23. *Combined corrections for cant and pitch*

directly with the weapon laying data. Rectangular coordinate equations, however, produce corrections that must be combined with linear target position quantities at some intermediate point in the process of computing the weapon laying data. In effect parallax corrections in general convert the target position as measured at the aiming device to the position of the weapon proper.

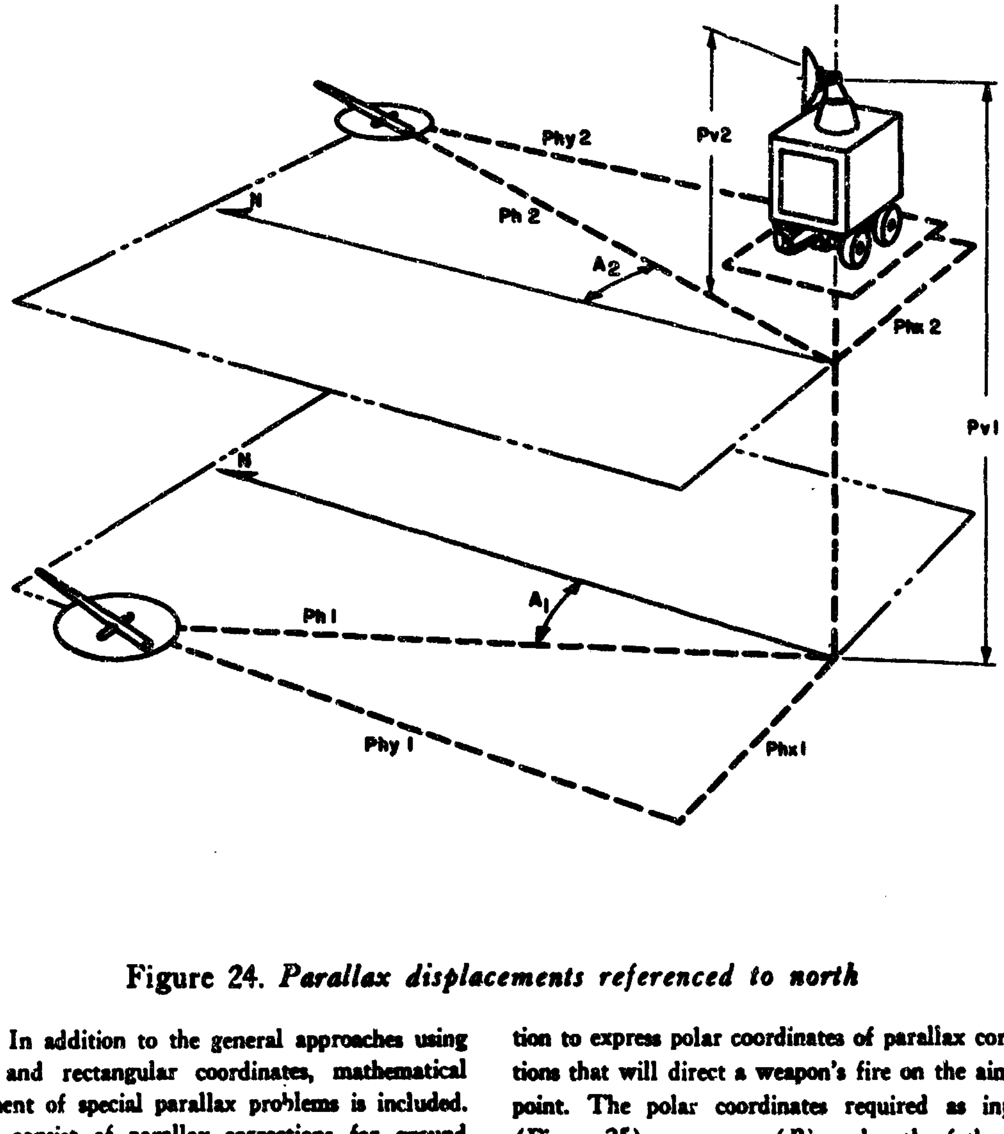


Figure 24. *Parallax displacements referenced to north*

c. In addition to the general approaches using polar and rectangular coordinates, mathematical treatment of special parallax problems is included. These consist of parallax corrections for ground targets under direct fire (on-carriage fire control) and an analysis of the parallax problem in the use of aiming posts for indirect fire.

19. Antiaircraft Problem in Polar Coordinates

a. Equations 22 through 26 employ polar coordinates of battery displacement and target location to express polar coordinates of parallax corrections that will direct a weapon's fire on the aiming point. The polar coordinates required as inputs (Figure 25) are: range (R) or length of the line of site of the aiming device; elevation angle of the aiming device line of site above the horizontal (E); vertical displacement of the aiming device above the weapon (vertical parallax base, Pv); horizontal displacement of the weapon from the aiming device (horizontal parallax base, Ph); and the horizontal azimuth angle (A) between the line Ph and the horizontal projection of the line of site.

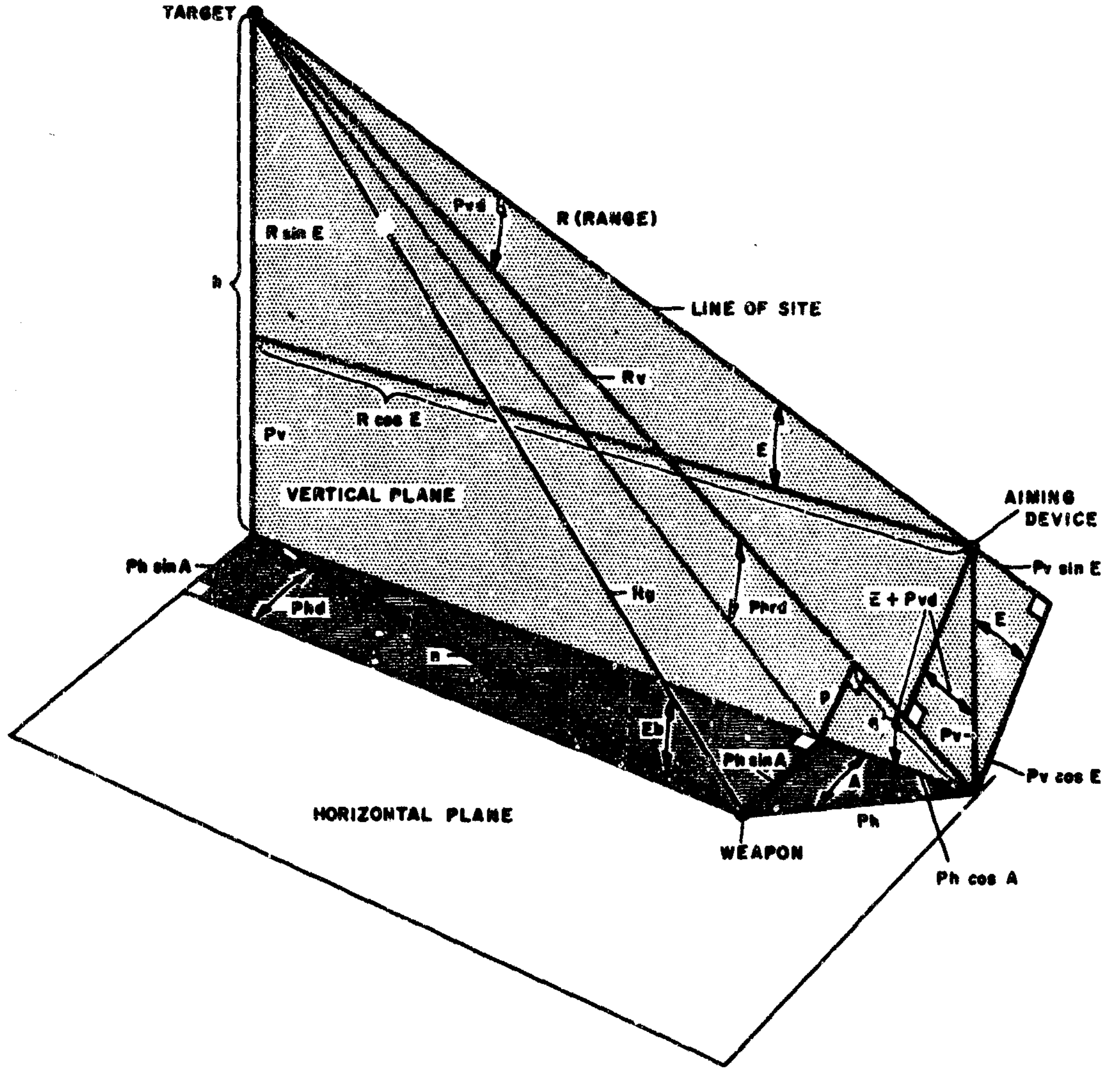


Figure 25. *Parallax corrections in polar coordinates*

b. The complete true solution requires a total of three parallax corrections: *Phd* for azimuth, *Pvd* for elevation, and *Prd* for range. In Figure 25, the azimuth parallax correction (*Phd*) is shown in the horizontal base plane as the angle necessary to converge the horizontal projection (*n*) of the weapon range (*Rg*) through the horizontal projection of the target. The other corrections, *Pvd* and *Prd* are not directly illustrated in Figure 25 because they are differences between quantities in separate, non-parallel planes. Specifically, *Pvd* is the difference between aiming device angle of site (*E*) and weapon angle of site (*Eb*). Also, *Prd* is the difference between the range from the aiming device to the target (*R*) and the range from the weapon to the target (*Rg*).

c. Additional equations for computing the elevation correction for vertical displacement, *Pvd*, and the elevation correction for horizontal displacement *Phrd* are included since partial corrections are sometimes useful in limited instruments. An equation for the partial elevation correction (*Prvd*) consisting of a combination of *Pvd* and *Phrd* is given in paragraph 22.b.(3).

d. The azimuth parallax correction to compensate for horizontal displacement, Ph, is:

$$\tan Phd = \frac{Ph \sin A}{R \cos E - Ph \cos A} \qquad \text{(Eq. 22)}$$

e. The total elevation correction for vertical and horizontal displacements is found by subtracting line of site elevation from an expression for the elevation of the target as viewed from the position of the weapon proper:

$$Ped = \text{arc}\tan\left\{\frac{(R \sin E + Pv) \sin Phd}{Ph \sin A}\right\} - E \qquad \text{(Eq. 23)}$$

f. The range correction is obtained by subtracting an expression for range from the weapon position, Rg, from line of site range, R:

$$Prd = R - Rg$$

Since Rg is the hypotenuse of the vertical right triangle connecting weapon and target,

$$Rg = \sqrt{n^2 + h^2}$$

In the horizontal plane,

$$\sin Phd = \frac{Ph \sin A}{n} \text{ and } n = \frac{Ph \sin A}{\sin Phd}$$

In the vertical plane of site,

$$h = Pv + R \sin E$$

Substituting:

$$Prd = R - \sqrt{\left\{\frac{Ph \sin A}{\sin Phd}\right\}^2 + (Pv + R \sin E)^2} \qquad \text{(Eq. 24)}$$

g. The elevation parallax correction to compensate for vertical displacement (Pv) alone may be computed from the following equation:

$$\tan Pvd = \frac{Pv \cos E}{R + Pv \sin E} \qquad \text{(Eq. 25)}$$

h. The elevation parallax correction to compensate for the range component of horizontal displacement ($Ph \cos A$) alone may be obtained as follows:

$$\tan Phrd = \frac{p}{Rv - q}$$

where:

$$p = Ph \cos A \sin (E + Pvd),$$

$$q = Ph \cos A \cos (F + Pvd),$$

and

$Rv = Pv \sin (E + Pvd) + R \cos Pvd$, or range corrected for vertical displacement (Pv) only

Substituting:

$$\tan Phrd = \frac{Ph \cos A \sin (E + Pvd)}{Pv \sin (E + Pvd) + R \cos Pvd - Ph \cos A \cos (E + Pvd)} \qquad \text{(Eq. 26)}$$

20. Field Artillery Parallax Corrections

Although Equations 22 through 26 are directly applicable to field artillery problems, they can be simplified considerably for this purpose because the elevation angles (angles of site) encountered are usually small enough to be neglected. By considering the angle E in Figure 25 to be equal to zero, Equation 22 becomes:

$$\tan Phd = \frac{Ph \sin A}{R - Ph \cos A} \qquad \text{(Eq. 27)}$$

Equation 23 becomes:

$$Ped = \text{arc} \tan \frac{Pv \sin Phd}{Ph \sin A} \qquad \text{(Eq. 28)}$$

Equation 24 becomes:

$$Prd = R - \sqrt{\left\{\frac{Ph \sin A}{\sin Phd}\right\}^2 + (Pv)^2} \qquad \text{(Eq. 29)}$$

Equation 25 becomes:

$$\tan Pvd = \frac{Pv}{R} \qquad \text{(Eq. 30)}$$

and Equation 26 becomes:

$$\tan Phrd = \frac{Ph \cos A \sin Pvd}{Pv \sin Pvd + R \cos Pvd - Ph \cos A \cos Pvd} \qquad \text{(Eq. 31)}$$

Equation 29 can be simplified further by dropping the $(Pv)^2$ term. This can be justified inasmuch as Pv is small in comparison to n making $(Pv)^2$ relatively insignificant.

21. Parallax Corrections in Rectangular Coordinates

a. In antiaircraft systems where gunnery computations are based on rectangular coordinates of the target's position, it is advantageous to determine and apply parallax corrections in terms of rectangular coordinates. The parameters remain fixed as long as the battery layout remains fixed, and the computations involve only simple algebraic additions. Although the case discussed here is based on an antiaircraft problem, the same principles can be applied to ground target fire control systems.

b. Figure 26 shows a single weapon in a battery and the target's location with respect to the aiming device initially determined by the spherical coordinates, range (R), elevation (E), and azimuth (A). During the course of computing lead angles, corrections, etc., the measured coordinates are converted to the rectangular coordinates, Xo, Yo, and Ho. Weapon position with respect to the aiming device established during emplacement is given in rectangular coordinates: Phx, Phy, and Pv. Since Phy is negative in the configuration of Figure 26, its addition to Yo makes Yg smaller than Yo. Combining the two sets of rectangular coordinates gives the coordinates of the target with respect to the weapon:

$$Xg = Xo + Phx$$

$$Xg = R \cos E \sin A + Phx \qquad \text{(Eq. 32)}$$

$$Yg = Yo + Phy$$

$$Yg = R \cos E \cos A + Phy \qquad \text{(Eq. 33)}$$

$$Hg = Ho + Pv$$

$$Hg = R \sin E + Pv \qquad \text{(Eq. 34)}$$

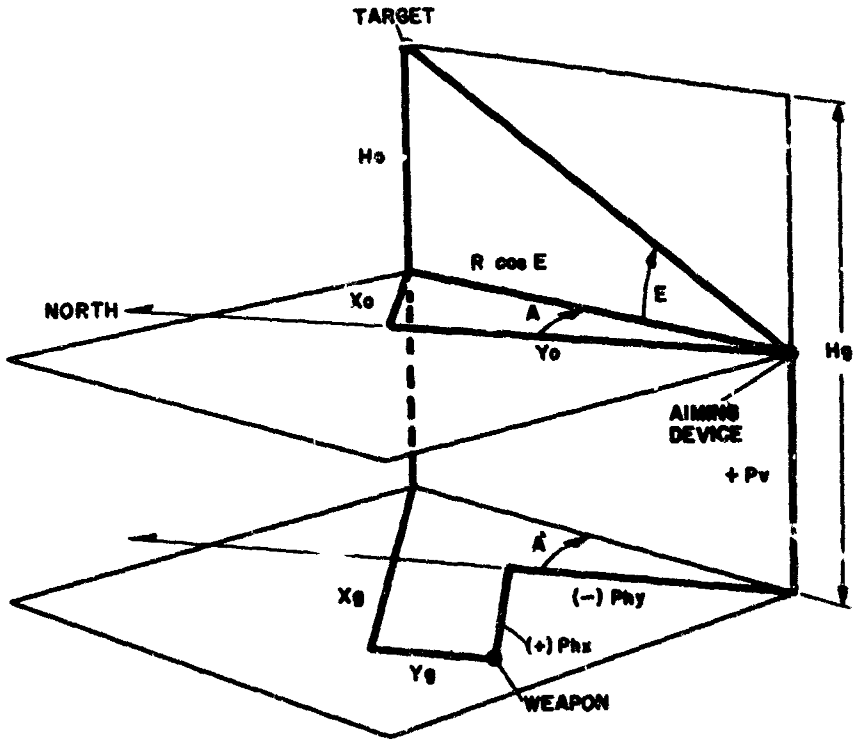


Figure 26. *Parallax corrections in rectangular coordinates*

22. Parallax in Direct-Fire Artillery

a. General. In high-accuracy direct-fire field artillery, the relatively small displacements between the weapon tube and the aiming device of the on-carriage fire control system can contribute intolerable errors to the azimuth and elevation settings of the weapon tube. The equations that follow are variations of the previously given antiaircraft polar coordinate equations (paragraph 19) especially adapted to the configuration of on-carriage fire control systems. Because the boresighting in weapons of this category may be based on either infinite or specific ranges, two sets of equations are given.

b. Infinity-Boresighted Systems. The solution of azimuth (*Phd*) and elevation (*Prvd*) correction equations requires knowns of line of site range (*R*) and elevation (*E*) and the rectangular coordinates of the displacement between the aiming device and the weapon proper. (See Figure 27). The complete elevation correction, *Ped*, includes compensation for the cross range component of horizontal displacement. The cross range component, however, is relatively small in on-carriage systems and contributes very little to the elevation parallax error. For this reason, an alternate elevation correction, *Prvd*, is presented, accounting for only the displacements in the elevating plane of the aiming device. The correction *Prvd* is actually a combination of the corrections *Ped* and *Phrd* (Equations 24 and 25 and Figure 25).

(1) The azimuth correction to compensate for displacement of the aiming device from the weapon bore can be calculated from the following equation (Figure 27):

$$\tan Phd = \frac{Phx}{R \cos E - Phr} \qquad \text{(Eq. 35)}$$

(2) The complete elevation correction for displacement between the weapon and aiming device can be found by subtracting the aiming device elevation (*E*) from an expression for the required weapon elevation (*Eb*):

$$Ped = \text{arc tan} \left\{ \frac{Pv + R \sin E}{d} \right\} - E$$

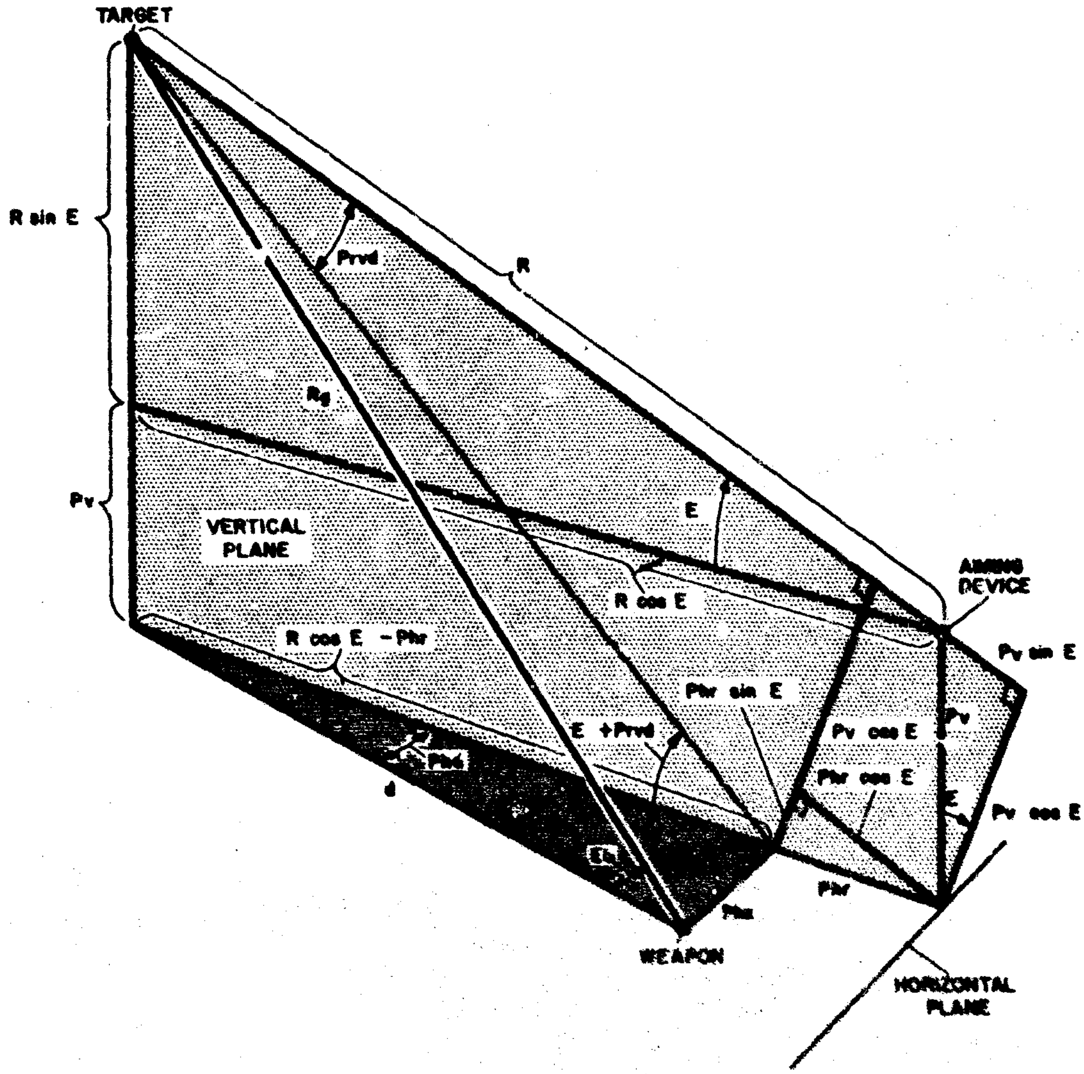


Figure 27. *Parallax in infinity-boresighted direct-fire systems*

$$d = \frac{Phx}{\sin Phd}$$

$$Ped = \arctan\left\{\frac{(Pv + R\sin E)\sin Phd}{Phx}\right\} - E \qquad \text{(Eq. 36)}$$

(3) The elevation correction to compensate for the horizontal and vertical displacements in the plane of line of site may be found from the following equation (Figure 27):

$$\tan Prvd = \frac{Phr\sin E + Pv\cos E}{R - (Phr\cos E - Pv\sin E)} \qquad \text{(Eq. 37)}$$

The numerator in the right half of Equation 37 represents the linear measure of the elevation parallax error.

c. Specific-Range Boresighted Systems. Two parameters not encountered in the infinity-boresighted system enter this problem: the range value, *R1*, which is the horizontal projection of the boresight range minus the range component of horizontal displacement, *Ph*, and angle *B*, the boresight depression angle. As in the case of the infinity-boresighted system described in the previous paragraphs,

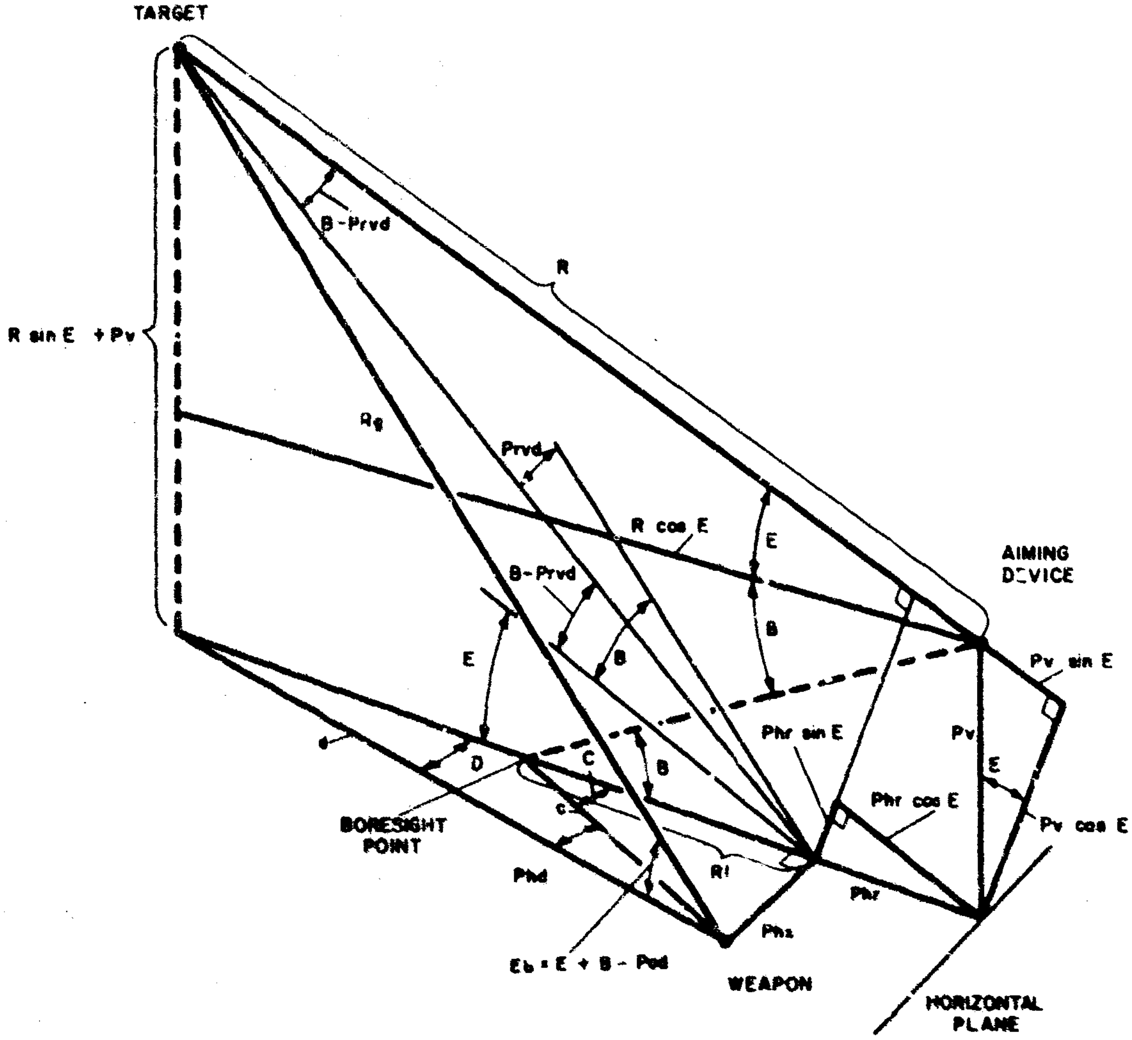


Figure 28. *Parallax in specific-range boresighted direct-fire systems*

an alternate elevation parallax equation is given which does not account for the negligible error contributed by the cross range component of horizontal displacement. The geometry of the problem is shown in Figure 28. This figure shows a boresight point on which the weapon bore is aligned initially at a horizontal distance d from the weapon. The weapon is displaced from the aiming device horizontally by the cross range component P_{hx} and the range component P_{hr}, and vertically by the distance P_v. In the initial alignment of the aiming device on the boresight point, the angles B and C are established. To engage the elevated target at range R, the system must be elevated through the angle $(E + B)$. To align the weapon bore on the target it must be elevated an amount differing from $(E + B)$ by the angle P_{vd}, and traversed to the left (in this case) through the angle P_{hd}.

(1) The elevation correction is the difference between an expression representing the required weapon elevation and the total aiming device elevation above the boresight point:

$$P_{vd} = \arctan \left\{ \frac{R \sin E + P_v}{d} \right\} - (E + B)$$

$$d = \frac{Phx}{\sin D} \qquad D = \text{arc tan}\left[\frac{R\cos E - Phr}{Phx}\right]$$

$$d = \frac{Phx}{\sin\left\{\text{arc tan}\left[\frac{Phx}{R\cos E - Phr}\right]\right\}}$$

$$Ped = \text{arc tan}\left\{\frac{(R\sin E + Pv)\sin\left\{\text{arc tan}\left[\frac{Phx}{R\cos E - Phr}\right]\right\}}{Phx}\right\} - (E + B) \qquad \text{(Eq. 38)}$$

(2) An alternate elevation correction that does not allow for the very small effect of cross range displacement (*Phx*) can be obtained (Figure 28).

$$Pvd = B - \text{arc tan}\left\{\frac{Phr\sin E + Pv\cos E}{R-(Phr\cos E - Pv\sin E)}\right\} \qquad \text{(Eq. 39)}$$

$$\text{where } B = \text{arc tan}\left[\frac{Pv}{R1 + Phr}\right]$$

(3) The horizontal parallax correction is the difference between the horizontal boresight angle C (Figure 28) and the required convergence angle D.

$$Phd = C - \text{arc tan}\left[\frac{Phx}{R\cos E - Phr}\right] \qquad \text{(Eq. 40)}$$

$$\text{where } C = \text{arc tan}\left[\frac{Phx}{R1}\right]$$

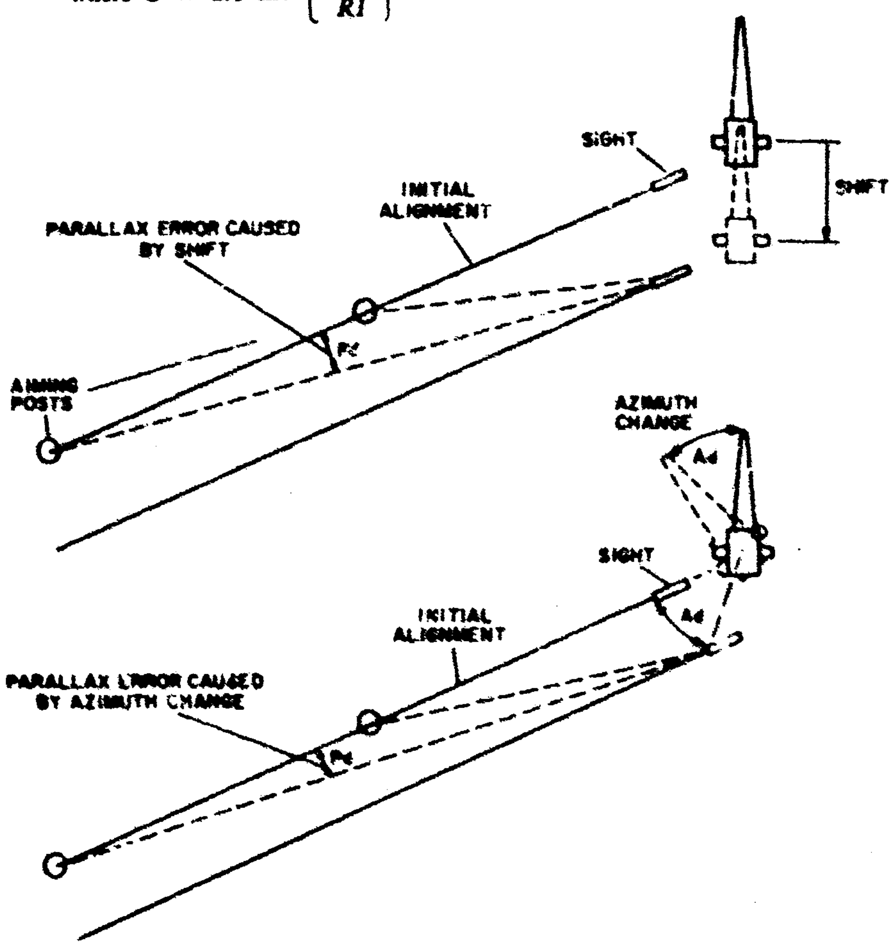


Figure 29. *Causes of parallax errors in aiming post alignment*

23. Parallax in Aiming Post Alignment

When aiming posts are used as a sighting reference for a weapon, errors are introduced whenever the weapon is changed in azimuth or the mount shifts from its initial position because of firing recoil. (See Figure 29.)

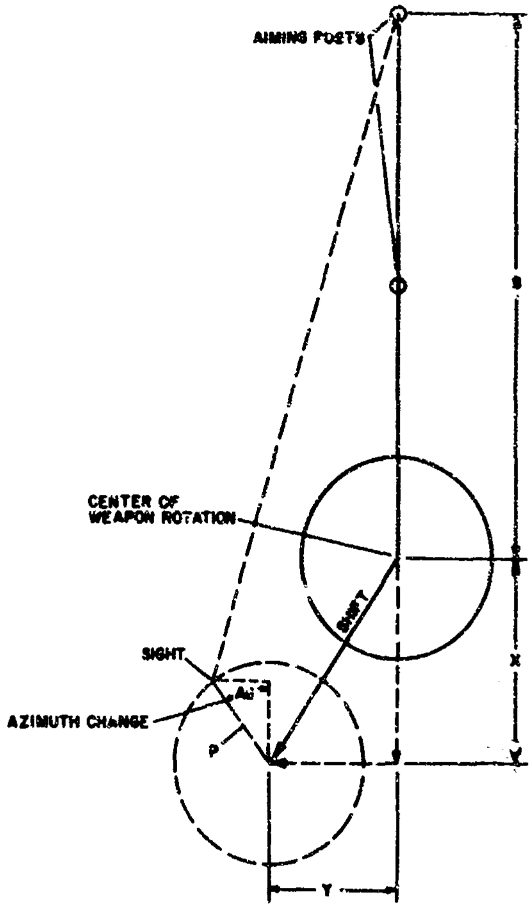
The method for correcting these errors has been by realignment procedures so that the initial parallax conditions are maintained. However, this procedure does not correct for the parallax error introduced.

a. The error introduced by an azimuth change in the weapon is given below. (See Figure 30.)

$$\tan Pd = \left| \frac{P \sin Ad}{S - P \cos Ad} \right| \qquad \text{(Eq. 41)}$$

b. The error introduced by an azimuth change and shift of the weapon is given below. (See Figure 30.)

$$\tan Pd = \frac{P \sin Ad + Y}{S + X - P \cos Ad} \qquad \text{(Eq. 42)}$$



$$\tan Pd = \frac{P \sin Ad + Y}{S + X - P \cos Ad} \qquad \text{(Eq. 42)}$$

Figure 30. *Parallax error in aiming post alignment caused by weapon shift and change in azimuth*

Section IV
INSTRUMENTATION

24. General

After the solution to a compensation problem has been obtained, it is necessary to instrument or mechanize the mathematical relationships so that the solution can be put to practical use. This section illustrates how the solution to a typical compensation problem might be instrumented. In addition, the instrumentation for some of the mathematical equations for the conditions described in Section III that have been used in actual fire control systems is given.

25. Instrumenting a Solution

To illustrate how an equation for a compensation problem might be instrumented and how the number of components can be minimized by approximating a true solution, the following example is given. The true solution equation used in this example is Equation 22 (Section III) and its approximation which is derived in Appendix C is Equation C18. Being tangent functions, the practical application of both of these equations is limited to values of E below approximately 85°. In antiaircraft systems capable of greater elevations, a device for limiting E would have to be provided.

a. A Possible Method for Mechanizing Equation 22

$$\tan Phd = \frac{Ph \sin A}{R \cos E - Ph \cos A}$$

A more convenient form for mechanizing the above equation is obtained as follows:

$$\tan Phd = \frac{\sin Phd}{\cos Phd}$$

$$\frac{\sin Phd}{\cos Phd} = \frac{Ph \sin A}{R \cos E - Ph \cos A}$$

or

$$Ph \sin A \cos Phd = (R \cos E - Ph \cos A) \sin Phd$$

The quantities A, E, and R are all measurable and are usually available in a fire control system for transmission, electrically or mechanically, to a computing device. In the schematic diagram showing the electromechanical instrumentation of Equation 22 (Figure 31) these three quantities are shown as mechanical inputs. The quantity Ph is a hand-set mechanical input and can be measured as the horizontal displacement between weapon and aiming device at the time of emplacing the equipment.

The quantities R and Ph position the wiper arms of a pair of potentiometers and so are converted into analog voltages. These voltages excite stator windings in separate electrical resolvers that are mechanically positioned by E and A. Combining cosine outputs of the two resolvers in a resistance network forms one factor of the right half of the equation and applying this to a resolver driven by Phd completes the right half of the equation. The left half of the equation is produced by another Phd resolver from the sine output of the A resolver. Combining the two halves of the equation in a resistance network results in an error signal which is amplified for controlling the Phd servomotor. The servomotor drives Phd toward its correct value at which point the outputs of the two Phd resolvers are equal and the error signal is zero. Should an input quantity change and unbalance the equation the motor will correct Phd simultaneously and maintain the loop error signal at null.

b. A Possible Method for Mechanizing Equation C18. Equation C18 is an approximate version of Equation 22.

$$Phd = \frac{K\ Ph \sin A}{R \cos E}$$

This equation can be computed in a divider loop from the outputs of two electrical resolvers (Figure 32). A saving of two resolvers over the true solution mechanization described above is effected. It will be noted that the principal difference in the schematics for the true and approximate solutions lies in the output section. The output section for the approximate solution receives inputs of $Ph \sin A$ and $R \cos E$ from two resolvers. The resistance network applies the constant, K, to $Ph \sin A$ and feeds it to a high-gain amplifier. The amplifier adjust-

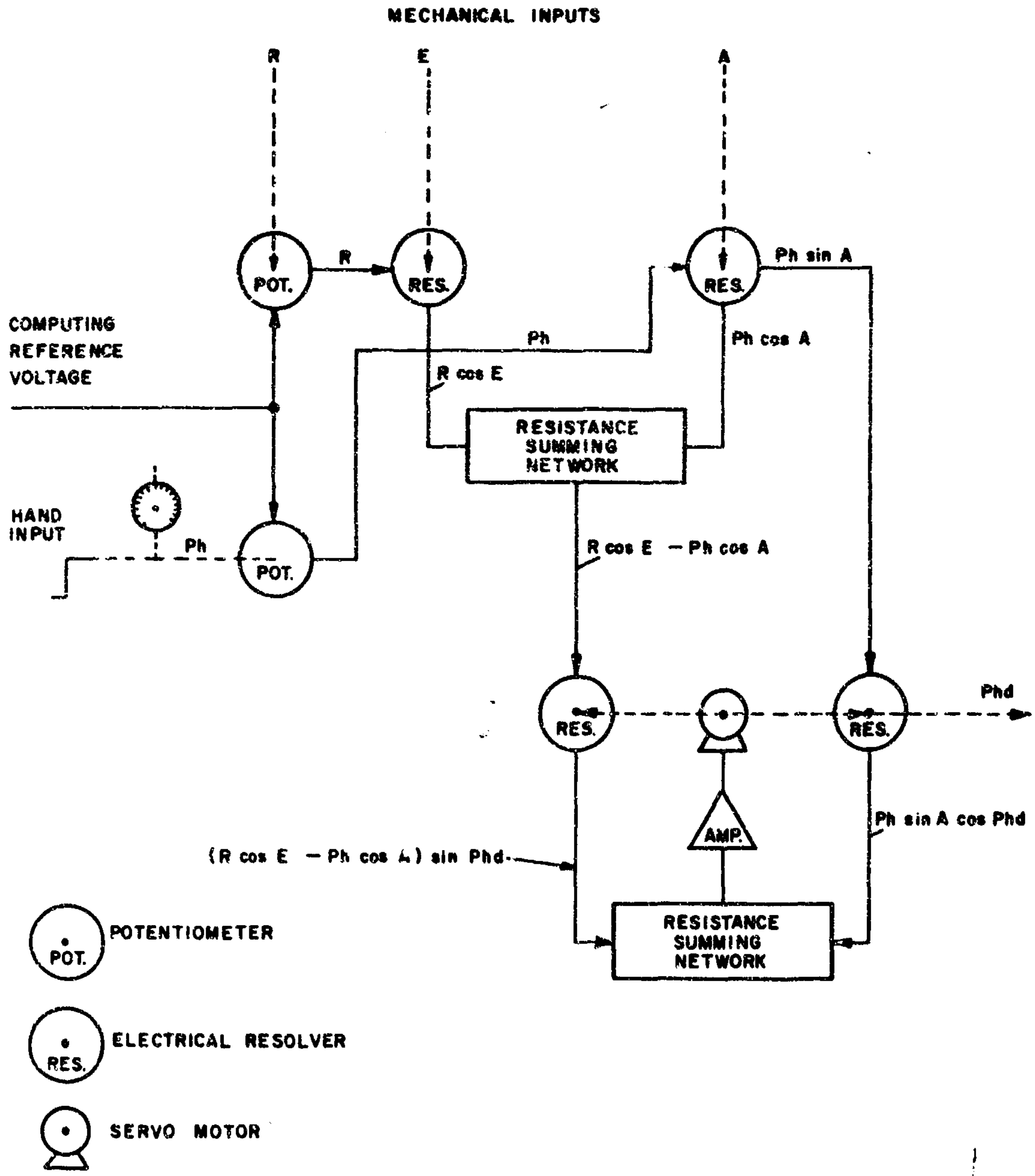


Figure 31. *Possible mechanization of Equation 22*

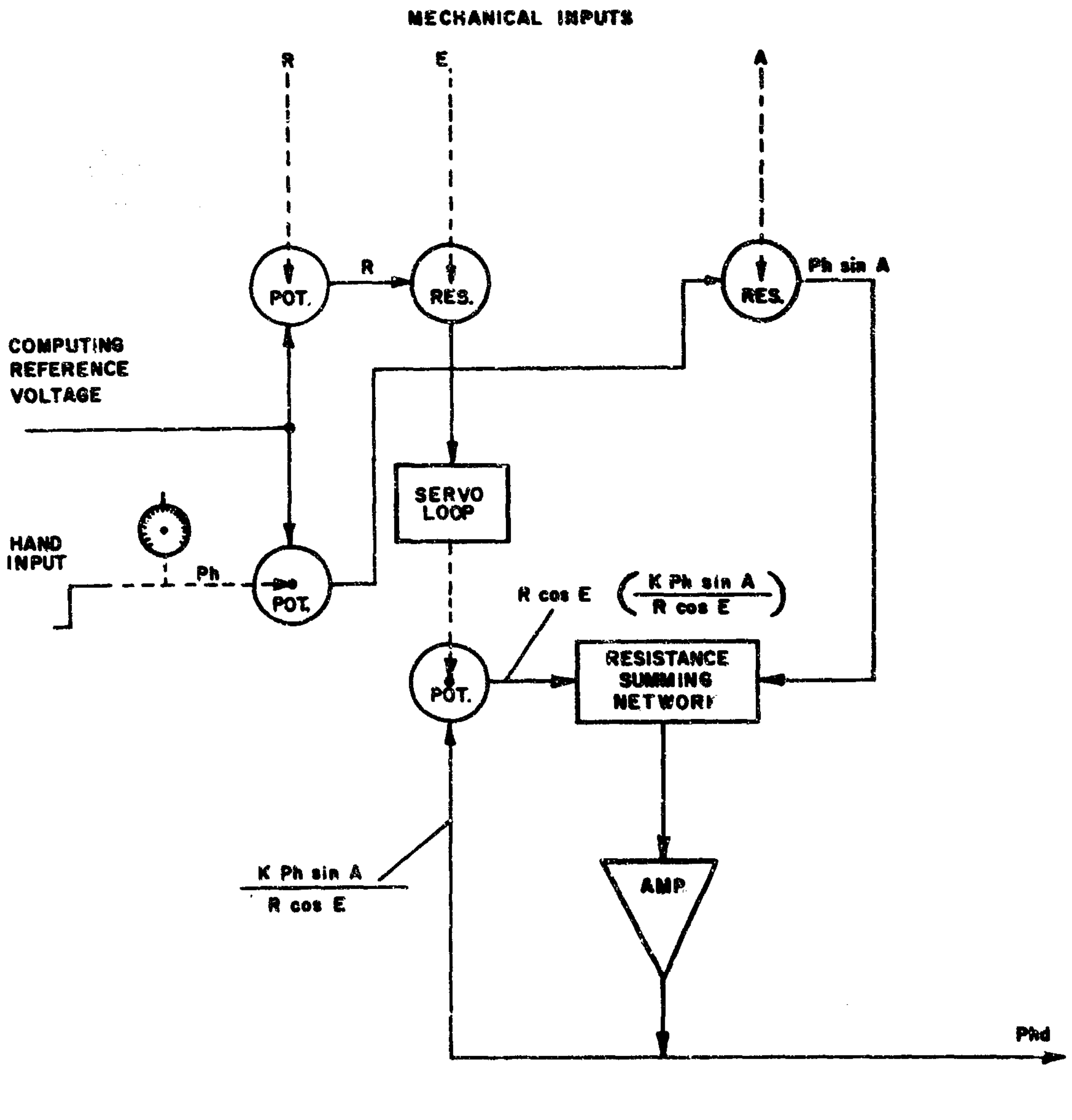


Figure 32. *Possible mechanization of Equation C18*

its output so that, when multiplied by $R \cos E$ in the potentiometer and fed back to the summing resistance network, it balances (nulls) the $Ph \sin A$ input of the network. To achieve this balance the amplifier output must assume the value of $\frac{K\ Ph \sin A}{R \cos E}$ which is equal to Phd. With a high-gain amplifier, the unbalance error signal required to produce a stable output and feedback is extremely small and can be considered a null.

26. Instrumentation in Typical Systems

The following paragraphs discuss some typical methods used at the present time in instrumenting the compensation of weapons for out-of-level and parallax conditions. The methods described are representative of both operational and experimental equipment.

27. Systems Using Off-Carriage Fire Control Data

This category covers field and antiaircraft weapons for which firing information is determined in a coordinate system other than the weapon's. The discussion on the cant-compensated telescope mount is stressed here because the heart of this device, a universal joint, is a true mechanical analog of the trunnion cant problem that has found widespread application in solving cant-compensation equations or portions thereof. The universal joint has been used in varied forms, both in simple telescope mounts as described below and as a computing element in complex electromechanical fire control systems.

28. Field Artillery (Indirect Fire)

a. Compensating Telescope Mount. The errors caused by trunnion cant are eliminated in principle when a sighting telescope is used with one of various types of compensated telescope mounts. Such a mount offsets the telescope line of site from true boresight alignment in proportion to the magnitude of the trunnion cant angle and the elevation angle of the weapon tube. Returning the line of site to the aiming reference by elevating and traversing the weapon introduces the corrections to the weapon tube.

(1) The key to the function of the compensated telescope mount is a Hooke's type universal joint. In its usual application, where it serves to couple two rotating shafts that are not in alignment, it appears as shown schematically in Figure 33A. Each shaft has a yoke supporting two bearings. A cross shaft whose arms (x and y) are assembled at a ninety-degree angle is carried in the four yoke bearings. For the condition illustrated in Figure 33A, where input shaft a is aligned with output shaft b rotation of the output will exactly equal the input throughout any portion of a revolution.

(2) In the compensated telescope mount, the universal joint is employed in the position shown in Figure 33B. The input shaft has been thrown ninety degrees out of alignment with the output shaft, at which point the output is zero for any degree of input rotation. On the weapon, the input axis a is set parallel to and rotated with the weapon trunnion. Axis y of the cross shaft is perpendicular to axis a and is set parallel to the weapon bore axis. Axis y therefore elevates with the weapon in a plane parallel to the plane of weapon elevation. Axis b carries the sighting telescope and is always maintained vertical, regardless of weapon tilt, by means of longitudinal leveling and cross-leveling adjustments provided. The x axis of the cross shaft will then be maintained horizontal. Sketch B shows the axes of the joint when trunnion cant (Cg) and weapon elevation (Eg) both are zero.

(3) In sketch C of Figure 33 an exaggerated cant angle (Cg) has been introduced, equally tilting the weapon trunnion (not shown) and the input shaft (a). The y cross shaft axis, known as the actuating arm, can no longer rotate in a vertical plane as shown in sketch B. If the weapon and actuating arm are elevated together (sketch D), the ends of the arm (axis y) will describe arcs that correspond to the arc ab in the front view in Figure 17 (Section III). With the elevation of the actuating arm confined to the canted plane, and axis x confined in a horizontal plane, axis b and its mounted telescope are driven through the angle Agd as the weapon tube and actuating arm are elevated through the angle Eq. Furthermore, axis x, confined to the horizontal plane, rotates about itself through the vertical quadrant elevation angle, Ea (arc cb, Figure 17). The spherical

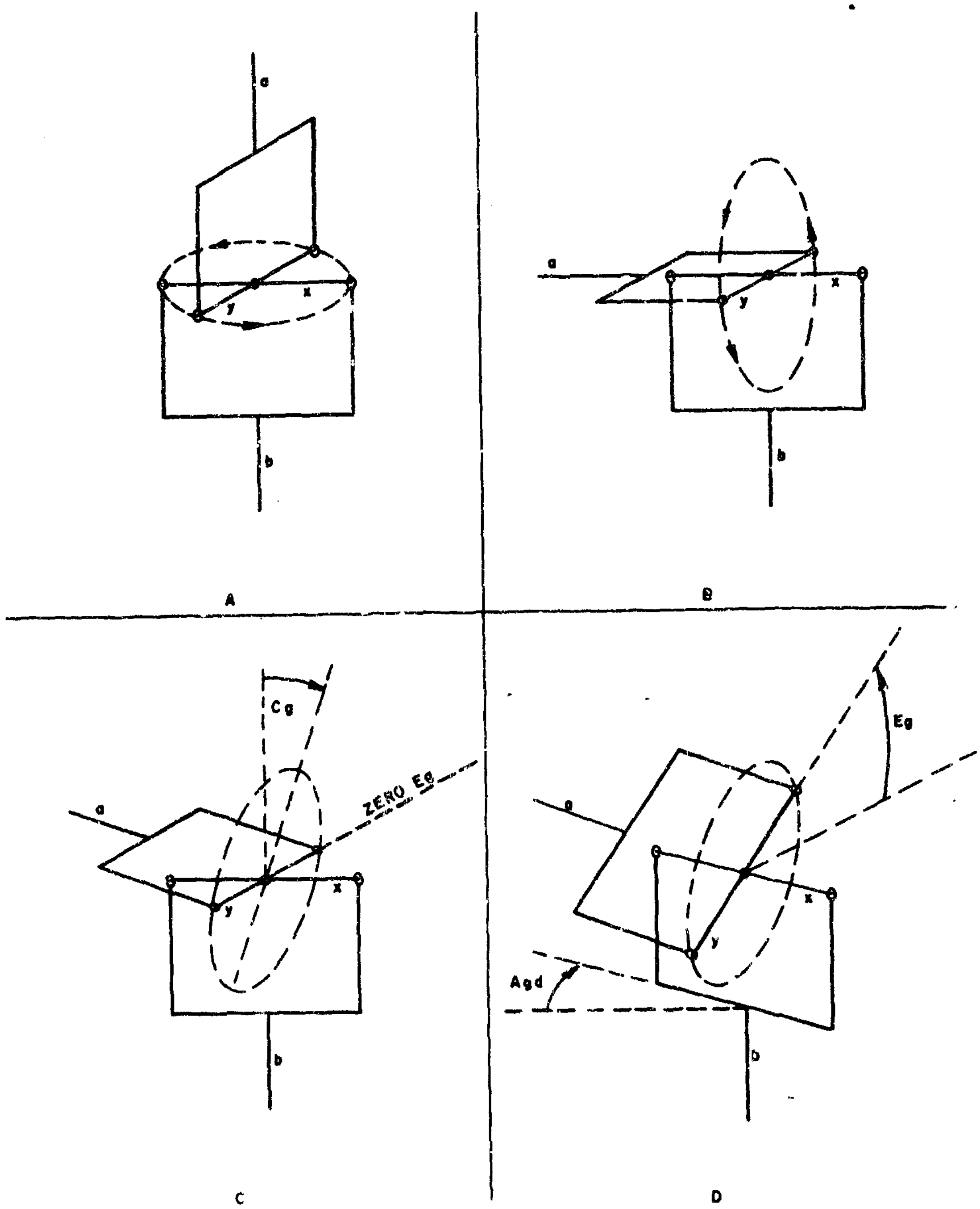


Figure 33. *Hooke's universal joint*

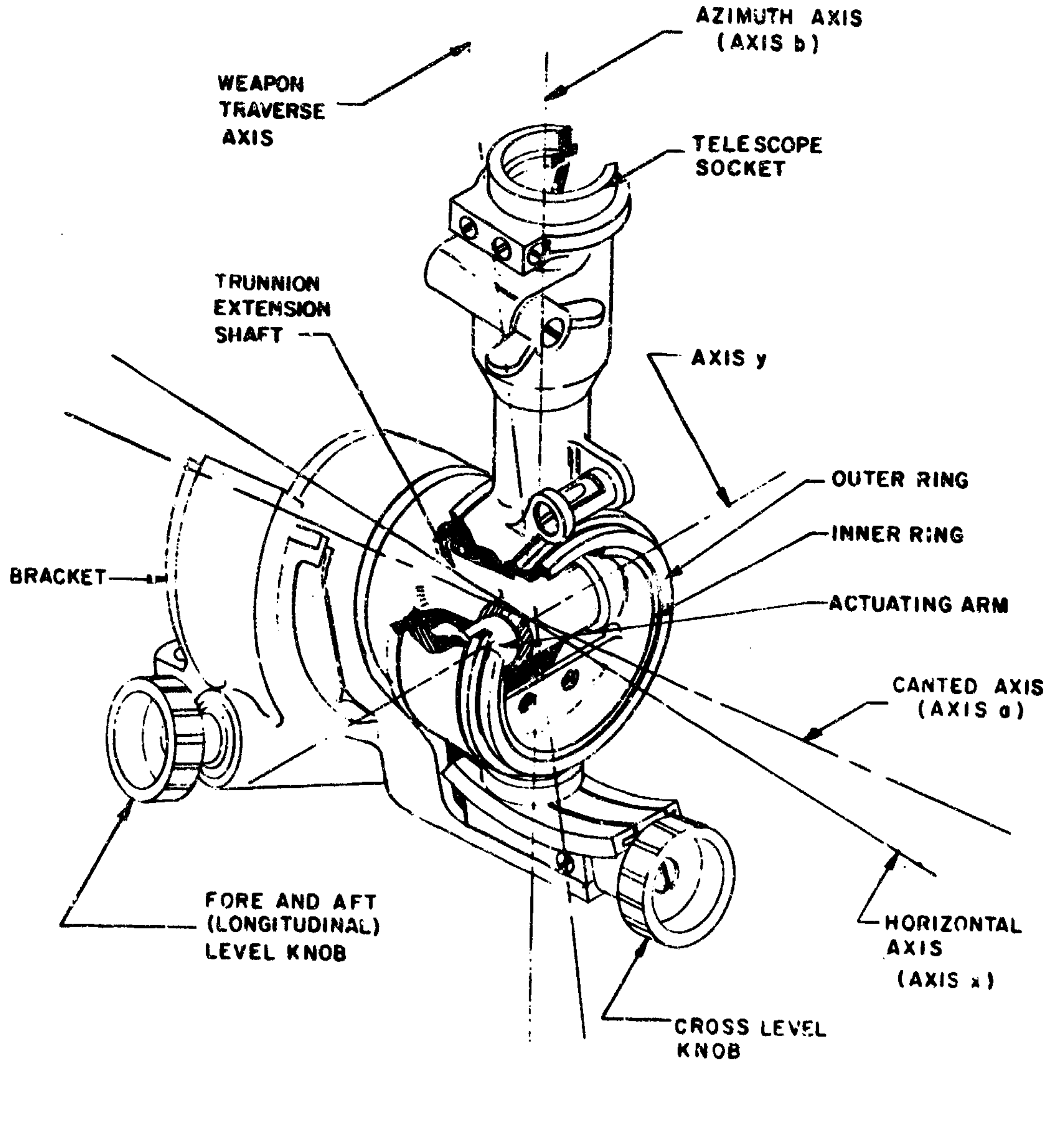


Figure 34. *Compensating telescope mount*

triangle of Figure 17 along with Equations 1, 2, and 3 therefore shows the mathematical relationship existing in the universal joint.

(4) In physical appearance, practical applications of Hooke's joint in compensating devices seldom resemble the ordinary shaft coupling device represented in Figure 33A. Its application in the common form of compensated telescope mounts is shown in Figure 34. In this form, the input yoke is omitted, the trunnion extension input shaft (axis a) being attached directly to the actuating arm (axis y). The x axis takes the form of a ring borne within another ring that substitutes for the output shaft yoke. The ends of the actuating arm pivot in bearings carried by the inner ring. The outer ring is equipped with a telescope socket at the top and is supported at the bottom by a pivot on the cross-leveling device. With the azimuth axis adjusted to the vertical position, (x axis horizontal) and the input axis from the trunnion canted, an elevation input will rotate the actuating arm in the canted plane. But the inner ring in which the actuating arm is pivoted is confined to rotation in the vertical plane about the x axis. This results in a component of the trunnion's rotation proportional to the sine of the cant angle being transmitted from the a axis to the b axis (azimuth axis). The offset thus produced is the azimuth error Agd which is corrected when the telescope axis is realigned on the aiming reference or target.

(5) Other applications of Hooke's joint take the form of a gimbal system in which the x axis is coupled to an indicator to show directly the value of quadrant elevation, Ea. When used as an element of a fire control computer, the device may not have any direct physical connection with the trunnion or aiming device. Instead, remote control via electrical or electromechanical inputs and outputs may be employed.

b. Range or Elevation Quadrant. To obtain true elevations that are not affected by the pitch of the weapon or by the cant of the trunnions, a range or elevation quadrant is used to lay the gun in elevation. The gunner's quadrant is leveled in the fore-and-aft and cross-level directions through the use of level vials. The fore-and-aft leveling eliminates the pitch error and establishes the zero elevation reference. The cross-leveling enables the gun elevation to be measured in a vertical plane. Elevation is measured in terms of elevation angle (mils) or range (yards).

c. Parallax Corrections.

(1) Aiming Post Shift.

(a) A primary method of establishing an azimuth aiming reference line for use in indirect fire with field artillery involves the use of a pair of aiming posts that are driven into the ground near each weapon. The most desirable distance from the weapon to the far aiming post (considering accuracy of laying, visibility, and ability to control aiming post lights when used) is 100 yards. The near aiming post is placed at the mid-point between the far aiming post and the weapon, and the near aiming post is aligned by the gunner so that the vertical reticle of the sight and the two aiming posts coincide. After the initial alignment, the near aiming post masks the far one.

(b) However, azimuth change of the weapon or progressive shifting of the mount from shock of firing or other effects causes the aiming posts to become misaligned. The vertical reticle of the telescope is displaced from the line formed by the two aiming posts and the posts appear to have shifted from their original position as shown in Figure 35.

(c) Corrections are then made manually by realigning the reticle so that the far aiming post appears halfway between the near aiming post and the vertical reticle. (See Figure 36.) This procedure maintains the line of site approximately parallel to the original line of site as shown in Figure 37.

(2) Movable Reticle.

One method that has been devised to facilitate the application of parallax compensation as described above involves the use of an additional vertical reticle that can be adjusted laterally with respect to a standard fixed reticle. The standard reticle is first aligned on the far aiming post by traversing the weapon. Then the vertical hairline of the movable reticle is moved into alignment with the near aiming post. The weapon is again traversed until the vertical hairline of the movable reticle is

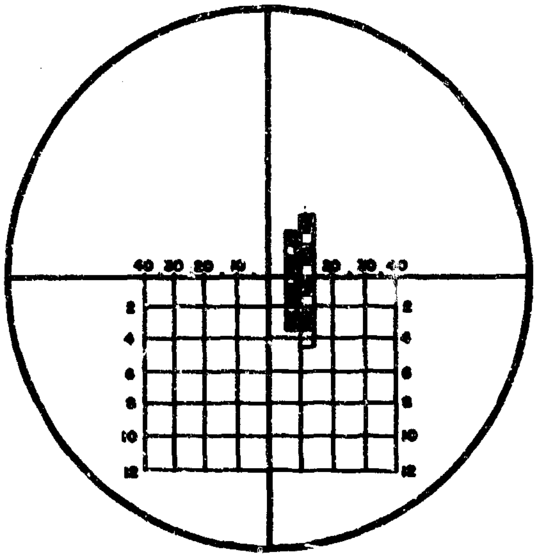


Figure 35. *Telescope axis not parallel to aiming posts after shift or azimuth change*

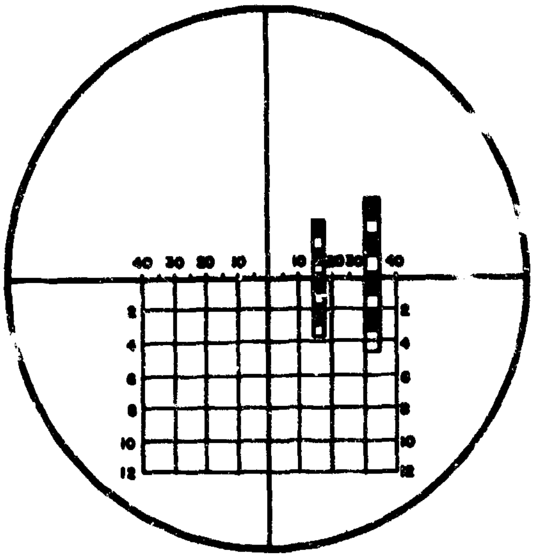


Figure 36. *Telescope axis aligned parallel with aiming posts*

aligned with the far aiming post, establishing the same conditions as are illustrated in Figures 36 and 37; i.e., the sight axis is parallel to the aiming reference line.

(3) Projection of Reticle at Infinite Range (Paralleloscope). Another method that has been used to keep a displaced line of site parallel to the original line of site for azimuth change or weapon shift is through use of the paralleloscope. The paralleloscope consists of a prism approximately 30 inches in length with a 2-inch face across ab (See Figure 38). The prism is placed approximately 15 to 22 feet away from the sight. The principle used in the paralleloscope is that the aiming point for the weapon sight is its own reflected image in the paralleloscope prism or the reflected image of a light beam projected from the sight. For any position of the sight there will be only one path where the transmitted light will coincide with the reflected light. Thus, when the sight is properly aligned, the transmitted light path and the reflected light path will be:

(a) Parallel to each other

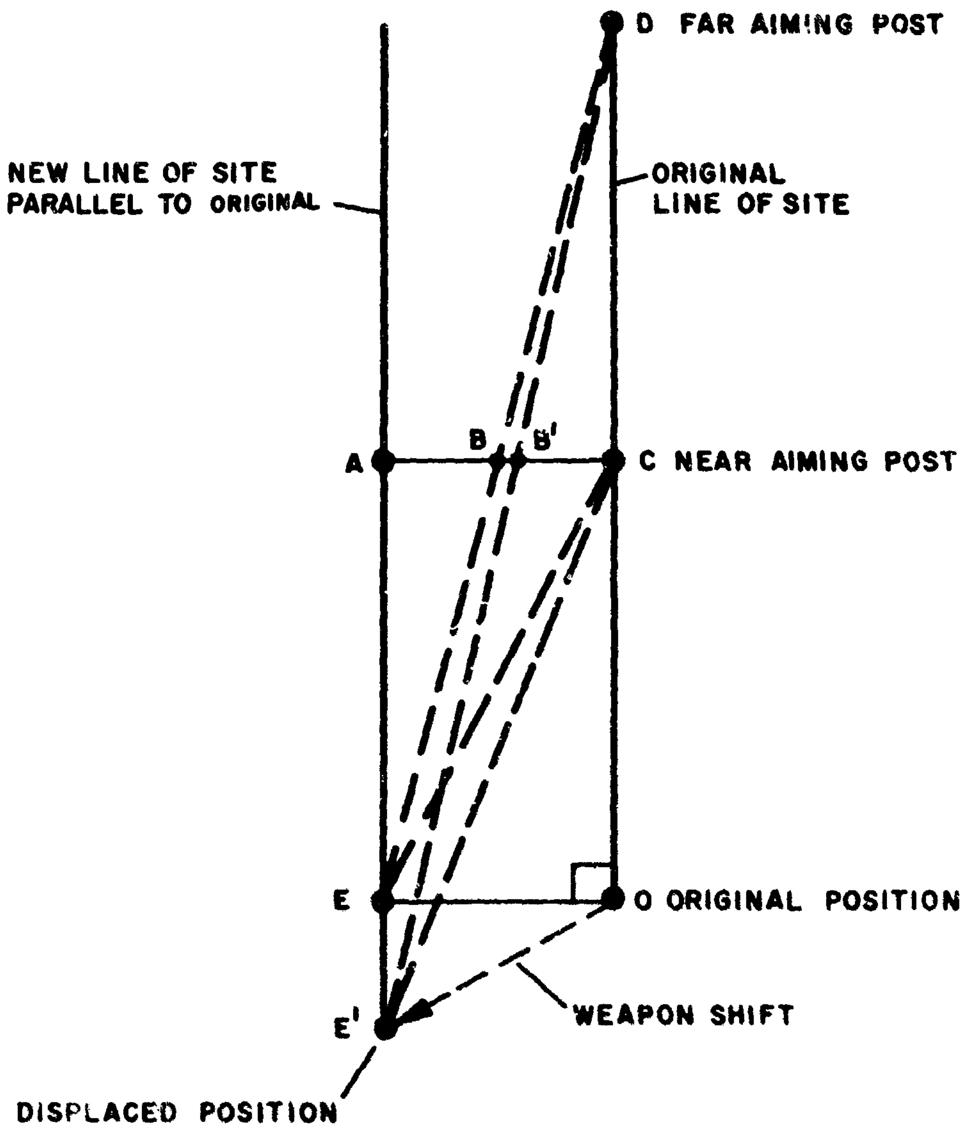
(b) Parallel to the initial line of site

(c) At 90 degrees with the face of the prism.

The length of the prism limits the amount of shift that the paralleloscope can accommodate before realignment is necessary.

(4) Computation of Battery Parallax Corrections (Plotting Board, Etc.). Firing data necessary for setting azimuth and elevation is usually obtained for the directing weapon of a battery only. Then, the data for the remaining weapons in the battery is generated at the fire direction center through use of plotting board, parallax tables, etc.

(5) Weapon Parallax Corrections. No special compensating devices have been employed for correcting parallax errors caused by displacement between the on-carriage aiming device and the weapon tube. Such small parallax errors are often



OC = CD

IF EE' = ZERO, AB = BC

IF EE' IS SMALL IN RELATION TO OD, AB' = B'C

Figure 37. *Geometry of aiming post shift problem*

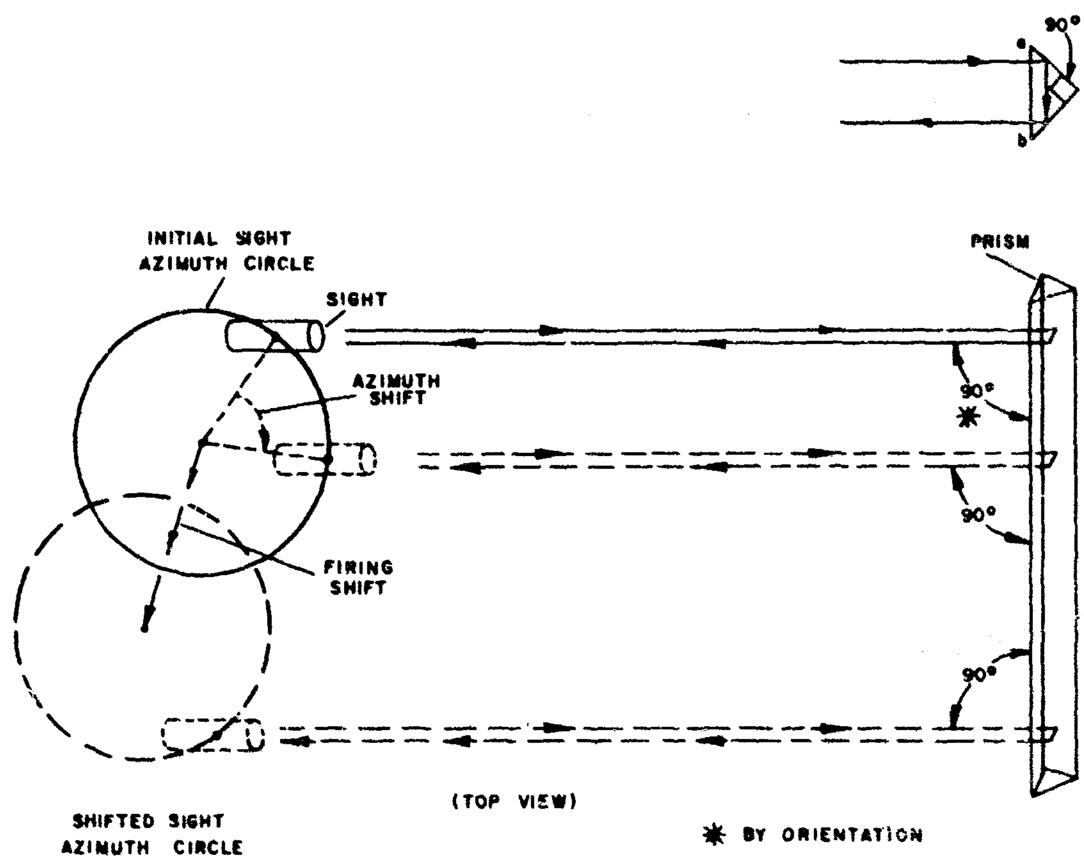


Figure 38. *Paralleloscope*

minimized by specific-range boresighting. Specific-range boresighting is the process of adjusting the sight axis of the aiming device and the gun bore axis of the weapon so that the two axes intersect at an aiming point, preferably within the range of employment of the weapon, but usually never less than the average range of employment. Parallax corrections can then be determined or computed from the aiming point reference.

29. Antiaircraft Artillery

a. Out-of-Level and Cant Corrections.

(1) Leveled Antiaircraft Mount. The most common method used to level AA weapons in the past has been with leveling jacks. Once the weapon has been leveled, the need for cant correction is eliminated. However, the inclusion of compensating elements in AA weapons simplifies emplacement, eliminates heavy leveling equipment, and increases mobility.

(2) Compensated Antiaircraft Mount. An experimental medium antiaircraft weapon was equipped with an automatic tilt corrector in order to reduce weight and improve mobility. The tilt corrector which eliminated the need for heavy, bulky, and cumbersome leveling jacks was lighter than the eliminated elements. The corrector is basically an electromechanical computer that transforms the azimuth (traverse) and elevation of the gun with respect to the gun mount into azimuth and elevation of the gun with respect to the level coordinate system of the director. The corrector utilizes mechanical inputs of actual gun traverse and elevation above the deck plane and electrical inputs of the required azimuth and elevation in the level

coordinate system transmitted from the director. The tilt of the gun mount is introduced manually in terms of a single tilt angle measured in a vertical plane through the gun bore axis at the azimuth position where the gun trunnions are level.

To complete the measurement for the out-of-level condition, the azimuth-of-tilt value is also introduced manually. Tilt and azimuth-of-tilt angles are determined and introduced during emplacement of the weapon and remain fixed until the weapon mount is again shifted. The geometry for this approach to out-of-level correction is shown in Figure 23 (Section III). From the varying inputs of gun traverse and gun elevation and the semifixed inputs of tilt and azimuth-of-tilt, the computer solves empirical equations based on Equations 20 and 21. The solution results in an azimuth correction and an elevation correction, which, applied to actual gun traverse and elevation, transforms these quantities to the level coordinate system.

The computed level coordinates of actual gun position are compared with the desired coordinates transmitted from the director. The comparison is carried out in the control transformer synchros that are used for receiving the azimuth and elevation transmission. When a difference exists, the control transformers deliver an electrical error signal to the gun drive causing it to change gun traverse or elevation as necessary to null out the error signal.

Essentially, this tilt corrector is an added link in the usual remote gun control system loop. It can be considered as a means of altering the loop feedback to balance out the inequalities between the coordinates of the level and out-of-level systems. This method of applying compensation is illustrated in Figure 39.

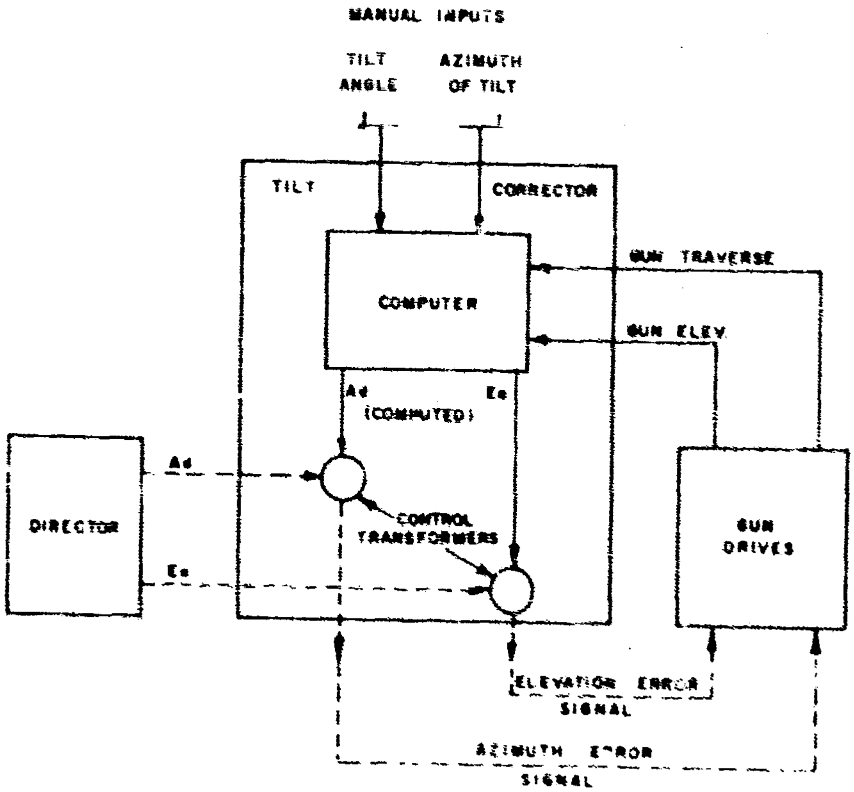


Figure 39. *Method of applying tilt compensation in feedback leg of gun control loop*

b. Parallax Corrections. In a typical antiaircraft battery of from two to four weapons and a director, the fire control computing system may be of the type that operates with rectangular coordinates of target position. The geometry of the parallax problem for this type of system was discussed in paragraph 21 and shown in Figure 26 (Section III). One method devised for combining the rectangular coordinates of target position with those of the parallax displacements is based on the use of voltage analogs to represent the target coordinates in an electrical computing system. The individual displacement coordinates are converted into voltages by means of potentiometers, as shown in Figure 40. These voltages are then algebraically combined with the target position coordinates by resistance networks. The resulting voltages represent the target's position with respect to a particular weapon.

30. Systems Using On-Carriage Fire Control Data

The following paragraphs give a general discussion of current and experimental methods for introducing corrections when the aiming device is located on the weapon mount and can be considered to operate in the same coordinate system as the weapon.

31. Tanks and Other Mobile Mounted Weapons

a. Cant Corrections.

(1) Compensating Periscope. A cant correction feature incorporated in the periscope receives superelevation data from a computer. The data are combined with the trunnion cant angle, de-

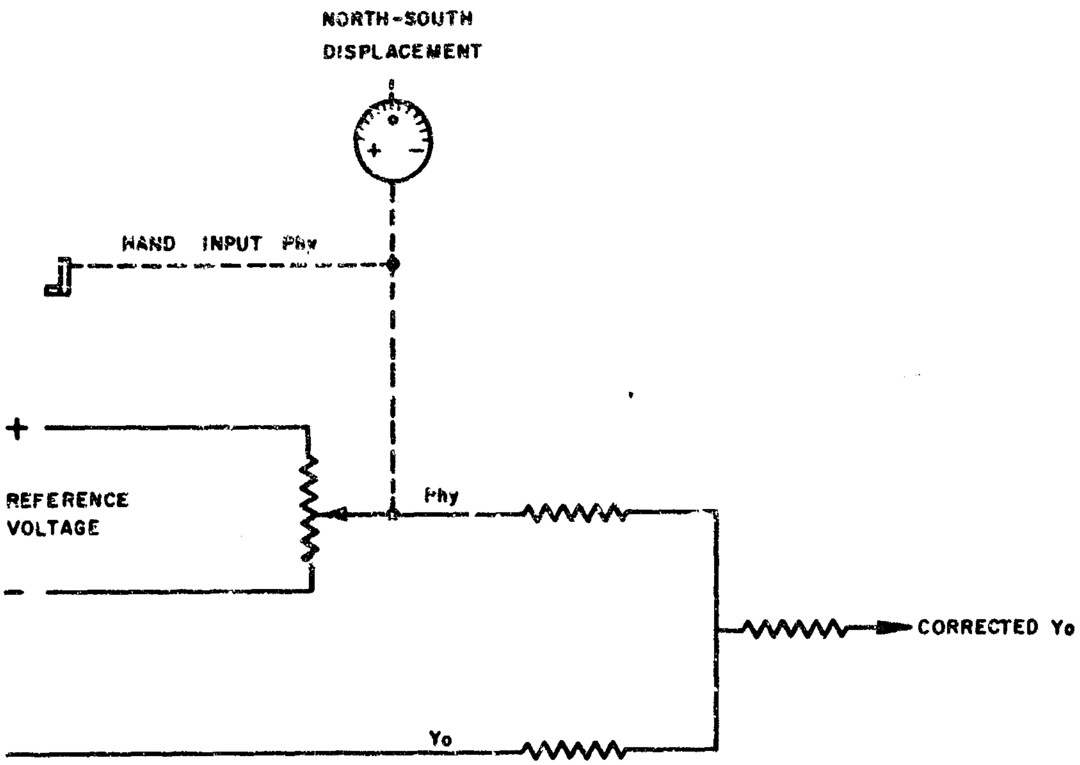


Figure 40. *Combing north-south weapon displacement with north-south target location data*

termined by manually leveling a vial on the body of the periscope. Then the output from the vial mechanism is fed to a corrector mechanism that displaces the reticle pattern within the periscope by an amount equivalent to the horizontal cant error.

(2) Rotating Telescope Reticle. This mechanism is similar to the compensating periscope in that the corrections are applied to the reticles of the sighting instrument. The telescope is equipped with a mechanism that enables the sighting reticle to be maintained in alignment with the level coordinate system, regardless of weapon tilt. Thus, ballistic corrections based on the level coordinate system can be introduced without error through normal application of the reticle markings. The relationships shown by Equations 13 and 14 (Section III) are satisfied in this mechanism. However, no computing elements, as such, are required. An oil-damped pendulum, serving as a vertical reference, derives an electrical error signal each time the tilt of the mount changes. The error signal requires no amplification since it is produced directly from the power source by means of contacts on the pendulum. A servomotor fed by the error signal drives until the reticle is erected, whereupon the contacts open and stop the rotation. A cross-leveling knob and vial are provided for emergency manual control of the reticle.

(3) Stabilizers for Moving Mounts.

(a) The gyrostabilizer is a device that assists the gunner in keeping the weapon of a tank properly laid as the tank moves over rolling terrain. The stable reference for this device is a gyro that in some designs also has been used to derive tracking rates for a lead angle computer. Relative motion of the tank about the stable gyro is picked off in two coordinates, either electrically or hydraulically. The tilt coordinates are then combined with target location and ballistic data to form composite gun orders for azimuth and elevation. These are transmitted to the azimuth and elevation hydraulic servomechanisms. In response, the weapon is positioned in the proper direction and at the same time moved in opposition to any longitudinal level or cross-level motion imparted to it by the tank hull.

(b) Another gyrostabilizing system that has been developed stabilizes the gun only in the elevation coordinate. This system is simpler, requires less power, yet provides a high degree of stability because the motion of a tank has a greater effect on elevation than azimuth. Operation of this system is similar to the system described in paragraph (a) above.

b. Parallax Corrections. To develop greater accuracy in direct fire, effort is made to eliminate the vertical parallax error caused by the relatively small vertical and horizontal displacements in the plane of fire. Equations 37 and 39 (Section III) show the relationships involved in true solutions for infinity boresighted and specific-range boresighted systems, respectively. No attempt has been made to instrument the true solution equations, but in one experimental tank, an inherent error in the superelevation drive gearing from the ballistic computer was given the appropriate sign to partially cancel the parallax error. This error in the superelevation gearing increased with increasing values of elevation in a manner similar to the vertical parallax error. It was due to planetary relative motion between portions of the superelevation gearing mounted on the elevating upper carriage and the portion emerging from the ballistic computer mounted on the nonelevating lower carriage. The feasibility of this approach was shown by comparing a tabulation of parallax errors computed from the equations with a parallel tabulation of the drive gearing errors. The comparison showed that the increase in vertical parallax error with increasing gun elevations could be practically compensated for at the most prevalent tactical ranges by the planetary error.

32. Antiaircraft (Self-Contained Mounts)

There are some weapon systems in use that have the fire control director mounted on the upper carriage of the weapon. One system of this type consists of a radar locator and tracker, an electromechanical computer that traverses and elevates with the gun. The system also contains an optical sighting system that can be used alone or in conjunction with the radar. The computer contains elements for correcting tilt of the weapon up to 8½°. The x and y components of tilt are set in during emplacement procedures by rotating two knobs on the computer. These knobs position potentiometers that derive voltages proportional to the x and y components of the out-of-level condition. The x any y values are combined with components of wind data and used with predicted firing azimuth, true elevation of gun tube, and time of flight of the projectile in computing a combined correction. The correction for tilt and wind is then combined with other ballistic corrections and applied to the firing data for positioning the gun tube.

33. Field Artillery (Direct Fire)

Direct fire, a secondary mission of field artillery, is essentially the same as for tanks. The methods for applying compensation in tanks (paragraph 31) are therefore adaptable to the direct fire control instruments of field artillery.

Section V
REFERENCE INFORMATION

34. Design Data

This section deals with the design of compensating elements from a broad point of view. The information included here encompasses general trends, an approach to error determination in misalignment, and general philosophical aspects of design.

35. Overall Accuracies

The overall accuracy of a compensating element is governed by type of weapon with which it will be used, and the tactical use of the weapon. In all types of weapons, design efforts have been toward greater firing accuracy. The accuracy of a compensating element is dependent, to a great extent, on the accuracy of the parts that go into it. It should be remembered, however, that even though overall accuracy requirements for a complete system or component may be highly stringent, it is still possible to produce a successful design using standard or even loose tolerances for the parts that compose the compensating element. This achievement is attained through the proper use of scale factors, and through the additive and subtractive effect of errors in a system.

36. Specific Accuracies

The principal problems of gun fire control are concerned with correcting weapon laying information. Even perfectly corrected firing data cannot be used to position a weapon in the correct direction and at the correct elevation unless the weapon has been properly manufactured, adjusted, and aligned with its aiming device. Some of the ideal conditions of alignment and adjustment are listed below. The conditions given are typical for a field artillery weapon using a compensated sight mount of the type described in paragraph 28, but the approach for other types of weapons is similar. A detailed analysis of the errors that result when the ideal conditions listed are not fulfilled is given in the paragraphs that follow. The conditions are:

a. The gun bore axis perpendicular to the trunnion axis.

b. The actuating arm (gun bar) axis parallel to the gun bore axis.

c. The actuating arm (gun bar) axis of rotation parallel to the trunnion axis.

d. Opposite arms of parallelogram linkages connecting the aiming device to the weapon trunnion of equal length.

37. Gun Bore Axis Not Perpendicular to Trunnion Axis

a. The relationships for conditions of misalignment when the gun bore axis is not perpendicular to the trunnion axis and cant is present, as illustrated in Figure 41, are:

$$\sin Ea = \cos\Delta I \cos Cg \sin Eg - \sin \Delta I \sin Cg \quad \text{(Eq. 43)}$$

$$\tan \delta I = \frac{\tan \Delta I \cos Cg}{\cos Eg} + \tan Eg \sin Cg \quad \text{(Eq. 44)}$$

where:

δI = Angle between horizontal projection of bore axis and horizontal perpendicular to trunnion.

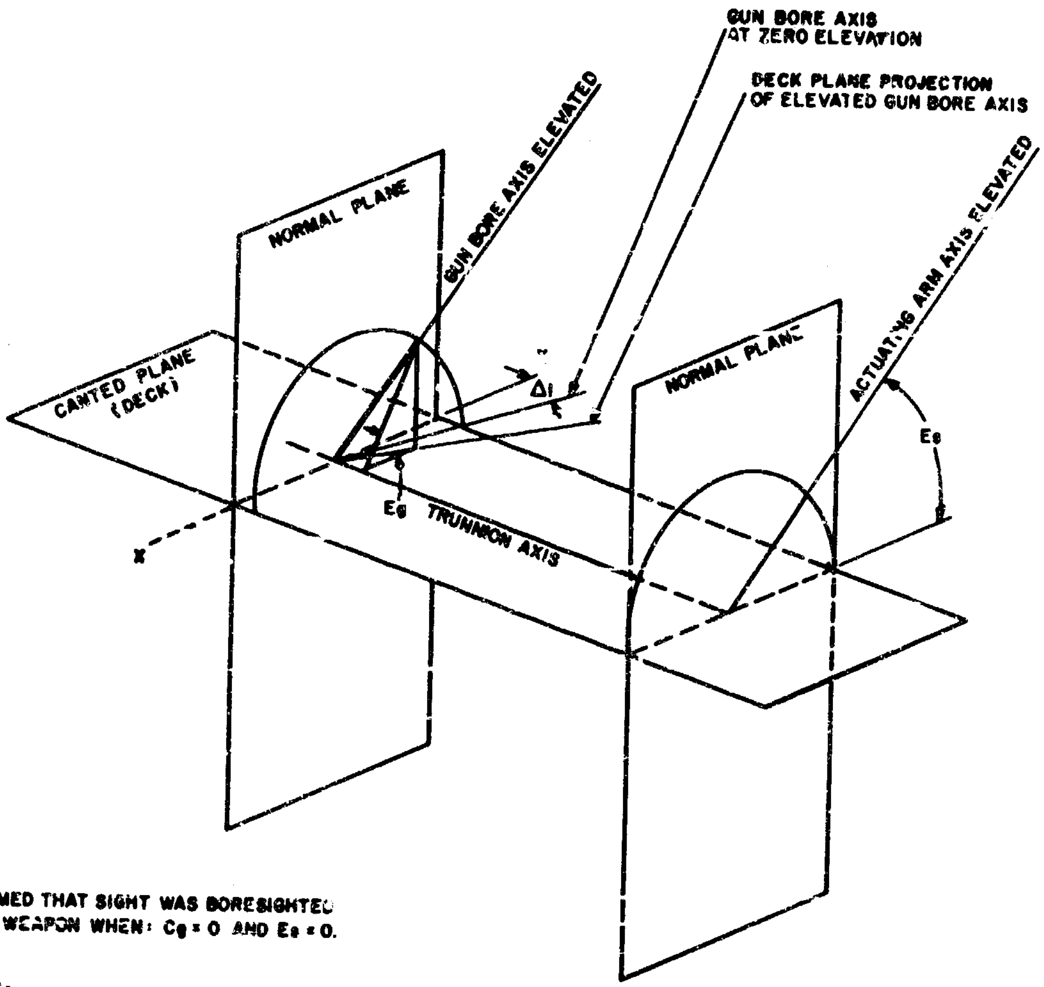
ΔI = Alignment error. Deck plane angle between bore axis ($Eg = 0$) and X-X axis.

This relationship holds also when an aiming device is trunnion mounted and the aiming device mount axis is not perpendicular to the trunnion axis. The same relationship applies to the case where the actuating arm (gun bar) axis of a linkage-driven sight is not perpendicular to its axis of rotation.

b. When cant is not present, Equations 43 and 44 can be simplified. (See Figure 42.) The equations for this condition are:

$$\sin Ea = \cos \Delta I \sin E \quad \text{(Eq. 45)}$$

$$\tan \delta I = \frac{\tan \Delta I}{\cos E} \quad \text{(Eq. 46)}$$



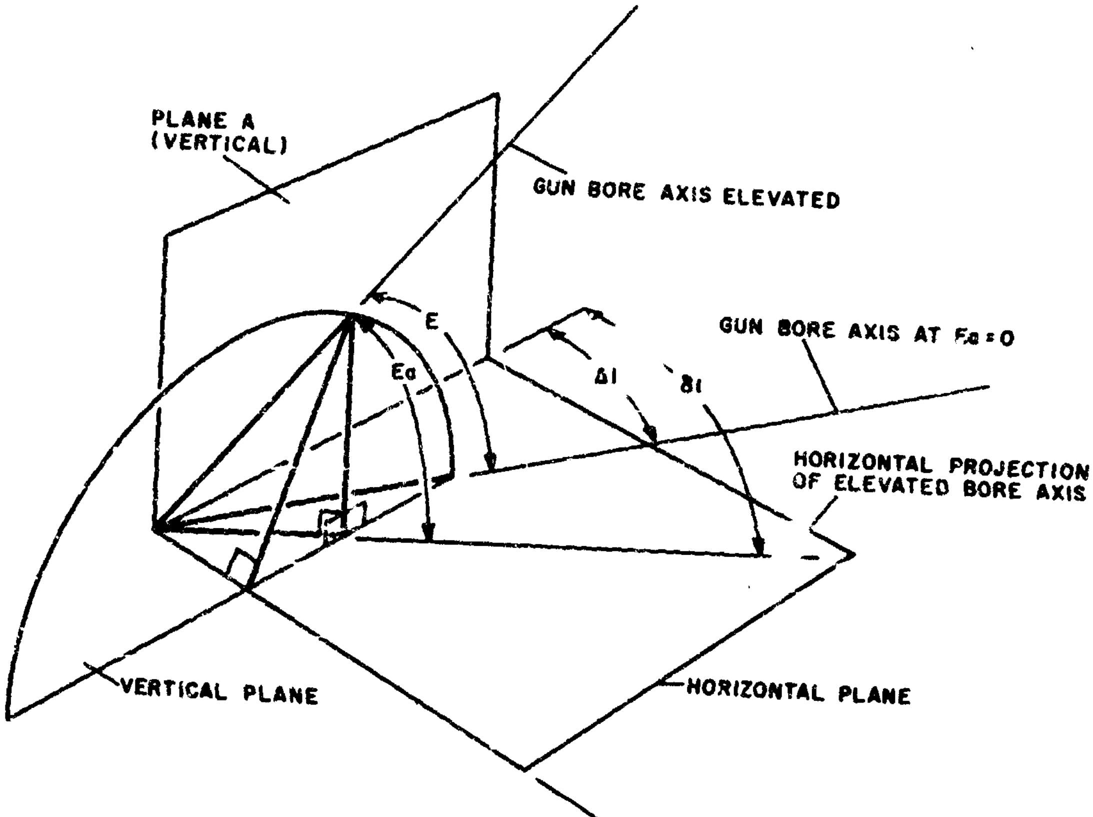
ASSUMED THAT SIGHT WAS BORESIGHTED
WITH WEAPON WHEN: $C_g = 0$ AND $E_a = 0$.

$E_g = E_a$

$$\sin E_a = \cos \Delta_1 \cos C_g \sin E_g - \sin \Delta_1 \sin C_g \qquad \text{(Eq. 43)}$$

$$\tan \delta_1 = \frac{\tan \Delta_1 \cos C_g}{\cos E_g} + \tan E_g \sin C_g \qquad \text{(Eq. 44)}$$

Figure 4[illegible]. *Gun bore axis not perpendicular to trunnion axis—trunnions canted*



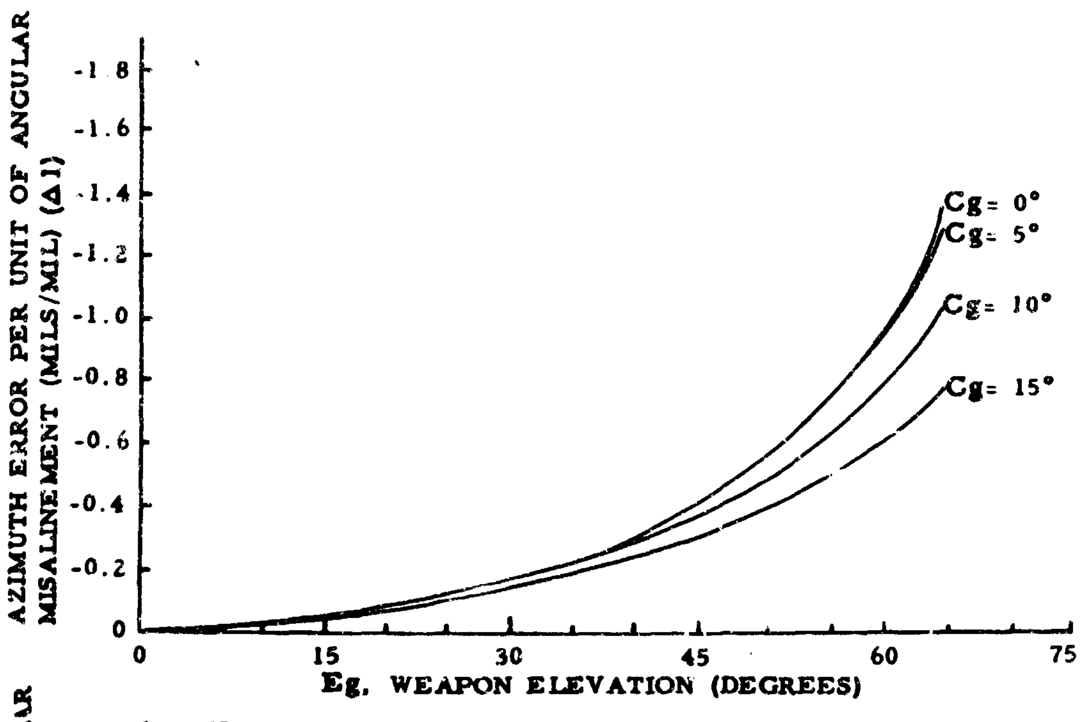
$$\sin E_a = \cos \Delta I \sin E \qquad \text{(Eq. 45)}$$

$$\tan \delta I = \frac{\tan \Delta I}{\cos E} \qquad \text{(Eq. 46)}$$

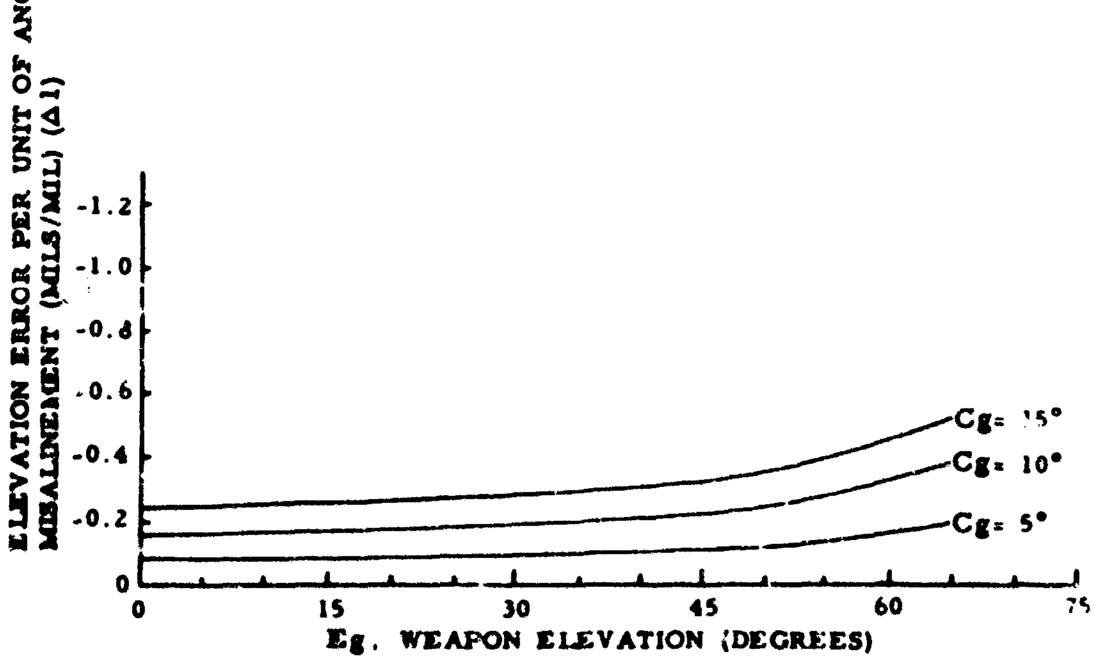
or

$$\cos \Delta I = \frac{\cos \Delta I \cos E}{\cos E_a} \qquad \text{(Eq. 47)}$$

Figure 42. *Gun bore axis not perpendicular to trunnion axis—trunnions level*



A. ERRORS IN AZIMUTH



B. ERRORS IN ELEVATION

Figure 43. *Plot of Error Equations 48 and 49*

Unit errors resulting from weapon tube axis not being perpendicular to the trunnion axis as a function of weapon elevation for various cant angles, Cg or

$$\cos \delta I = \frac{\cos \Delta I \cos E}{\cos Ea} \quad \text{(Eq. 47)}$$

c. If the involved angles of the situation shown in Figure 41 are confined to small values, the following equations show approximately the relationship between the weapon position error and the alignment error.

$$\frac{\text{Azimuth Error}}{\Delta 1} = \left\{ \frac{\cos Cg\ (1 - \tan^2 Eg \sin^2 Cg)}{\cos Eg} - 1 \right\} \quad \text{(Eq. 48)}$$

$$\frac{\text{Elevation Error}}{\Delta 1} = \frac{\sin Cg}{\sqrt{1 - \sin^2 Eg \cos^2 Cg}} \quad \text{(Eq. 49)}$$

It has been assumed that the aiming device was boresighted with the weapon when the cant angle (Cg) and weapon elevation (Eg) equalled zero.

d. Equations 48 and 49 are shown plotted in Figure 43 for different degrees of cant angle.

38. Actuating Arm (Gun Bar) Axis Not Parallel to Gun Bore Axis

a. The equations for error in azimuth and elevation when the actuating arm (gun bar) axis and gun bore axis are not parallel, as shown in Figure 44, are:

$$\frac{\text{Azimuth Error}}{\Delta 2} = \frac{\sin Cg}{1 - \sin^2 Eg \cos^2 Cg} \quad \text{(Eq. 50)}$$

$$\frac{\text{Elevation Error}}{\Delta 2} = \frac{\cos Ea \cos Cg}{\sqrt{1 - \sin^2 Eg \cos^2 Cg}} \quad \text{(Eq. 51)}$$

b. Equations 50 and 51 are shown plotted in Figure 45 for different values of cant angle.

39. Actuating Arm (Gun Bar) Elevating Axis Not Parallel to Trunnion Axis (Error Measured Parallel to Canted Plane)

a. The equations for error in azimuth and elevation for an aiming device mount that is linkage-driven and in which the actuating arm (gun bar) axis of rotation is not parallel to the trunnion axis, as shown in Figure 46, are:

$$\frac{\text{Azimuth Error}}{\Delta 3} = -\frac{\sin^2 Eg \sin^2 Cg \cos Cg}{1 - \sin^2 Eg \cos^2 Cg} \quad \text{(Eq. 52)}$$

$$\frac{\text{Elevation Error}}{\Delta 3} = -\frac{\cos Eg \sin Cg}{\sqrt{1 - \sin^2 Eg \cos Cg}} \quad \text{(Eq. 53)}$$

It has been assumed that the aiming device was boresighted with the weapon when the cant angle and weapon elevation equalled zero.

b. Equations 52 and 53 are shown plotted in Figure 47 for values of cant angle.

40. Actuating Arm (Gun Bar) Elevating Axis Not Parallel to Trunnion Axis (Error Measured Perpendicular to Canted Plane)

a. The equations for error in azimuth and elevation for an aiming device mount that is linkage-driven and in which the actuating arm (gun bar) axis of rotation is not parallel to the trunnion axis, as shown in Figure 48, are:

$$\frac{\text{Azimuth Error}}{\Delta 4} = \frac{\sin Es \cos Es \cos Cg}{1 - \sin^2 Es \cos^2 Cg} \quad \text{(Eq. 54)}$$

$$\frac{\text{Elevation Error}}{\Delta 4} = -\frac{\sin Es \sin Cg}{\sqrt{1 - \sin^2 Es \cos^2 Cg}} \quad \text{(Eq. 55)}$$

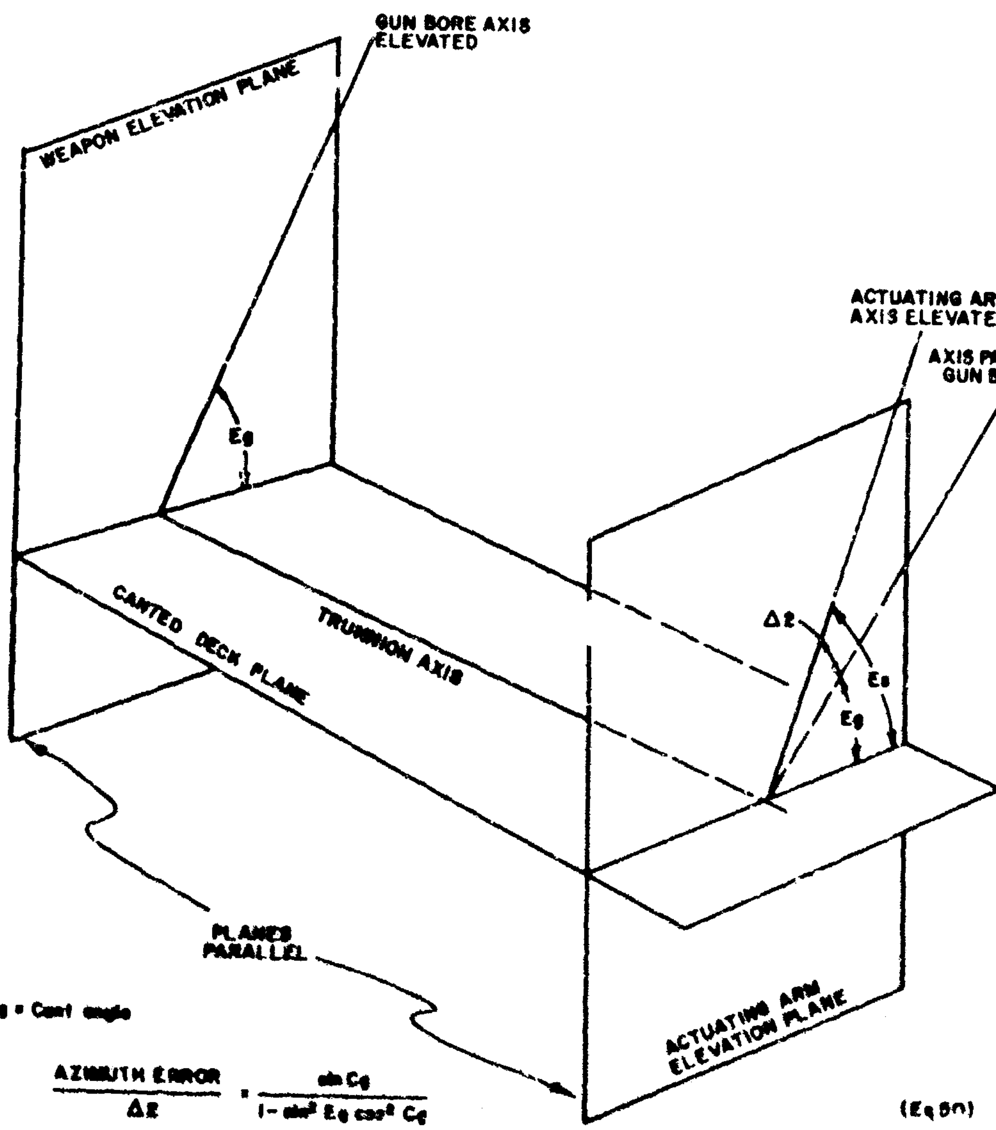
b. Equations 54 and 55 are shown plotted in Figure 49 for different values of cant angle.

41. Unequal Length in Parallel Linkages Between Aiming Device and Weapon

The equations for errors resulting from unequal lengths in the links used in parallelogram linkages between the aiming device and weapon, as shown in Figure 50, are:

$$\text{Error (mils)} = -\left(\frac{\Delta L}{L} \times 1000\right) \csc F \quad \text{(Eq. 56)}$$

$$\text{Error (mils)} = -\left(\frac{\Delta R}{R} \times 1000\right) \cot F \quad \text{(Eq. 57)}$$

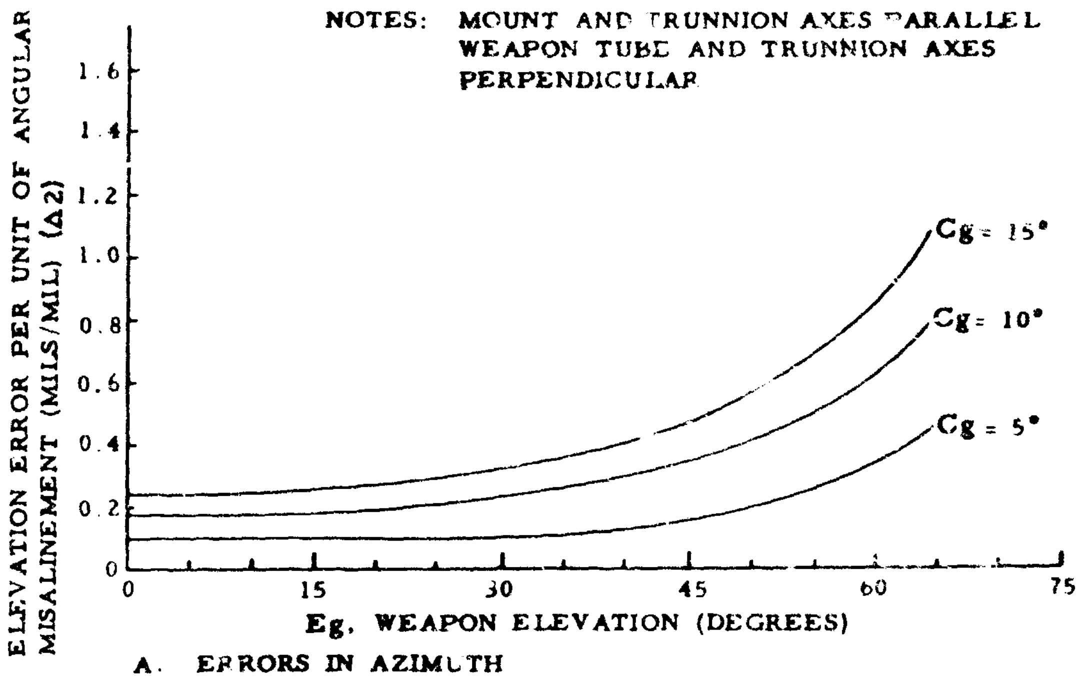


Cg = Cant angle

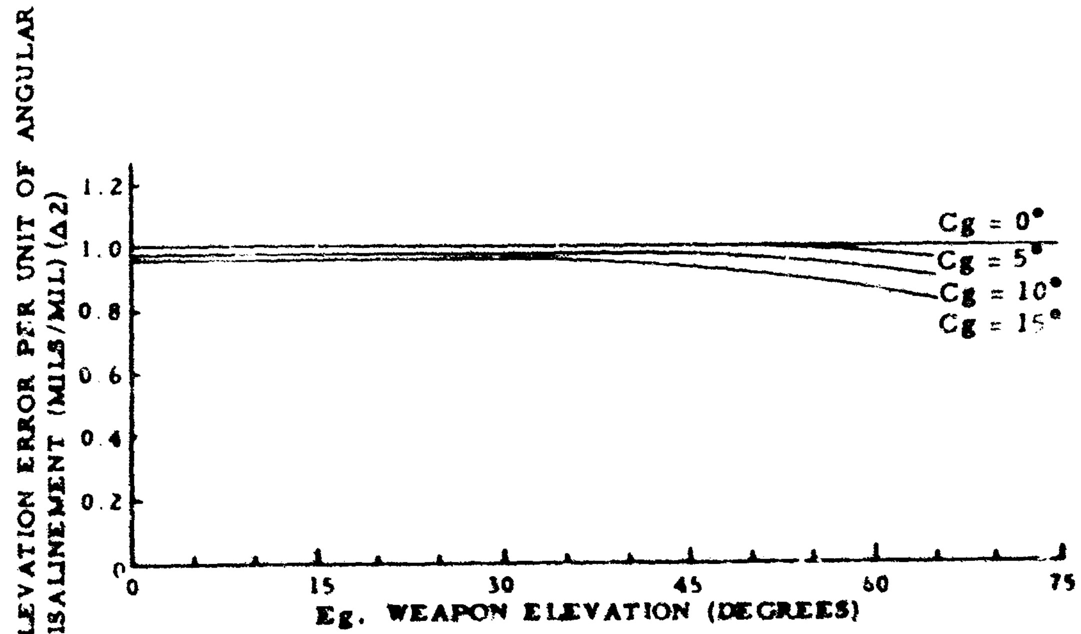
$$\frac{\text{AZIMUTH ERROR}}{\Delta z} = \frac{\sin C_g}{1 - \sin^2 E_g \cos^2 C_g} \qquad \text{(Eq 50)}$$

$$\frac{\text{ELEVATION ERROR}}{\Delta z} = \frac{\cos E_g \cos C_g}{\sqrt{1 - \sin^2 E_g \cos^2 C_g}} \qquad \text{(Eq 51)}$$

Figure 44. *Actuating arm axis not parallel to gun bore axis*

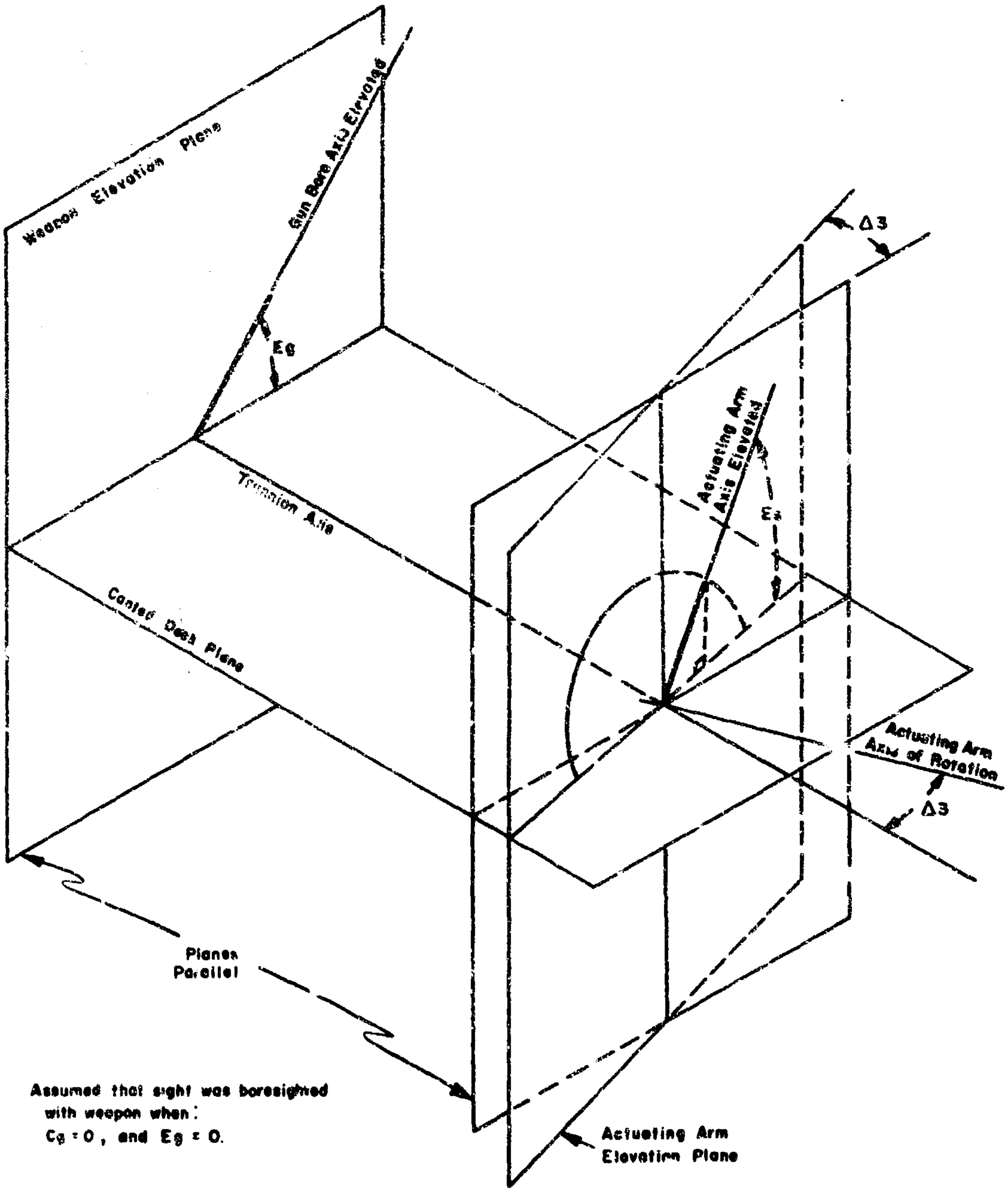


A. ERRORS IN AZIMUTH



B. ERRORS IN ELEVATION

Figure 45. *Plot of Error Equations 50 and 51*
Unit errors resulting from elevation angle deviation between weapon tube and mount gun bar axes as function of weapon elevation for various cant angles, Cg



$$\frac{\text{Azimuth Error}}{\Delta_3} = -\frac{\sin^2 Eg \sin^2 Cg \cos Cg}{1 - \sin^2 Eg \cos^2 Cg} \qquad \text{(Eq. 52)}$$

$$\frac{\text{Elevation Error}}{\Delta_3} = -\frac{\cos Eg \sin Cg}{\sqrt{1 - \sin^2 Eg \cos Cg}} \qquad \text{(Eq. 53)}$$

Figure 46. *Actuating arm elevating axis not parallel to trunnion axis (error in canted plane)*

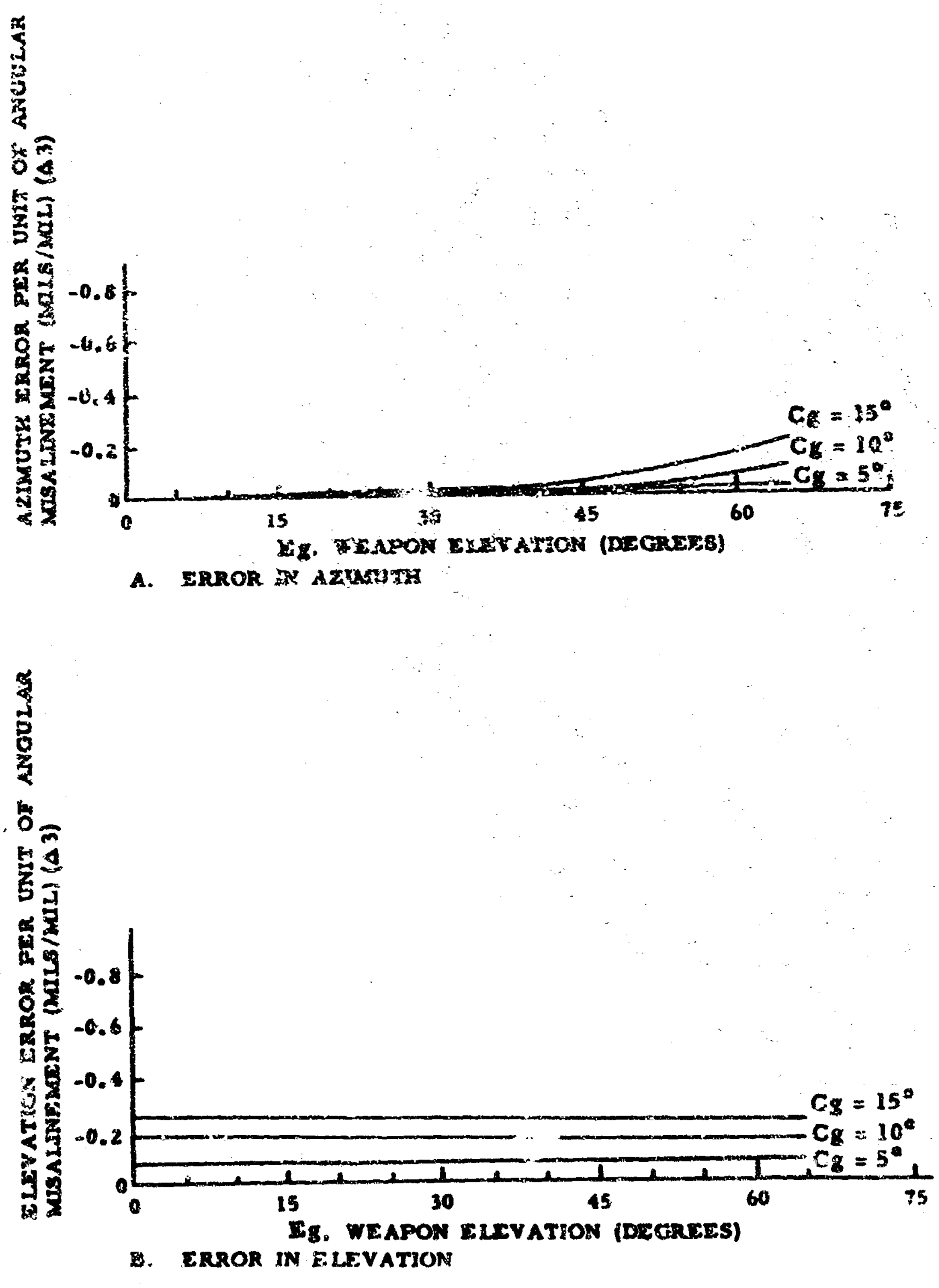
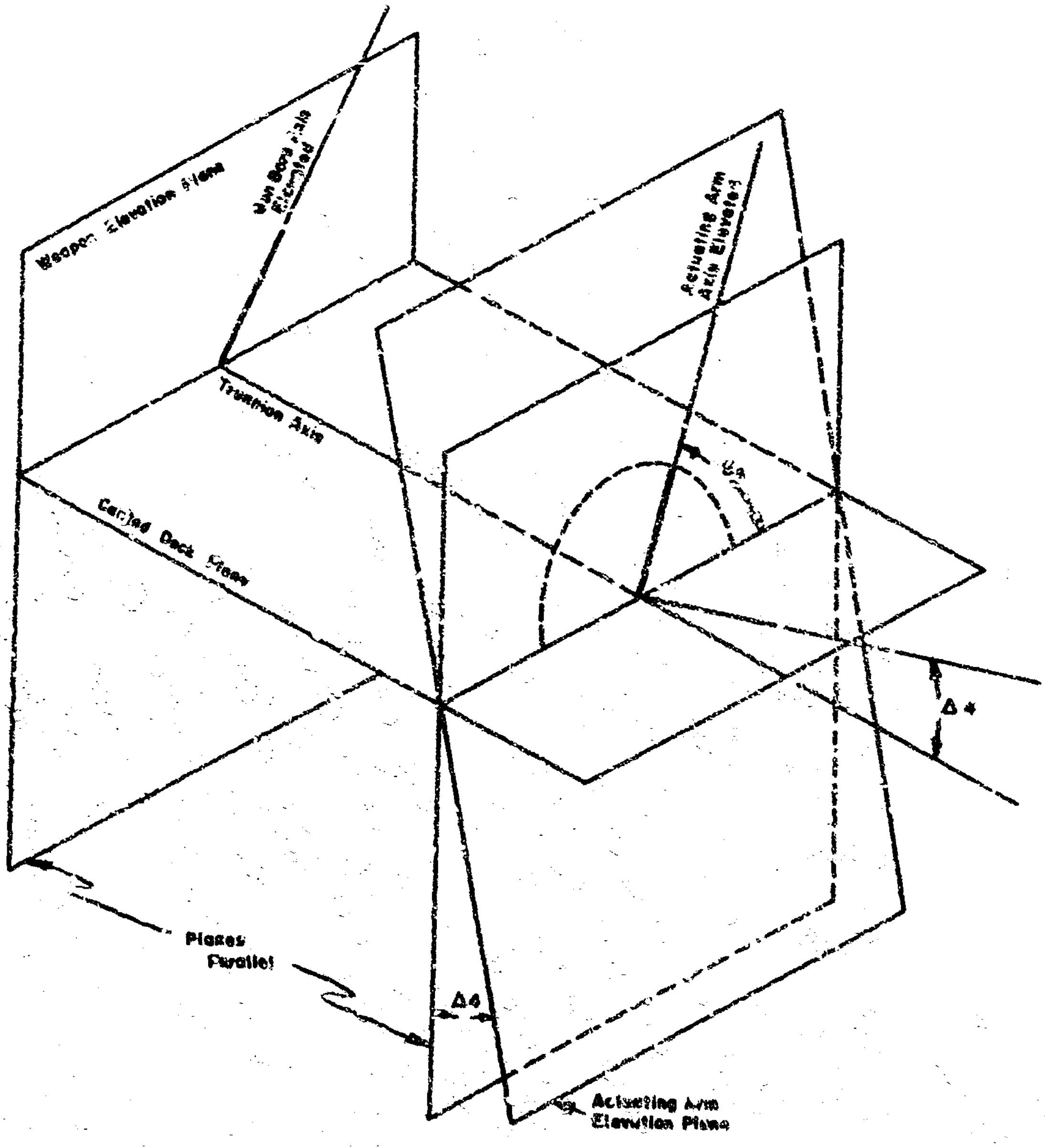


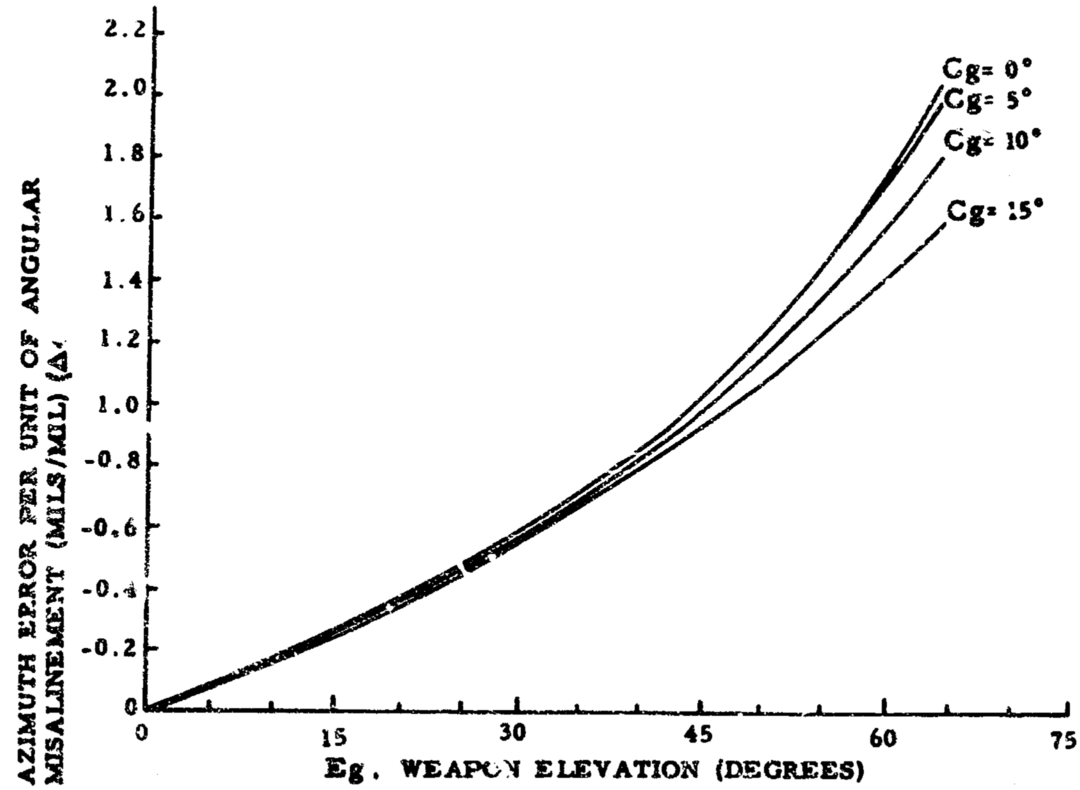
Figure 47. *Plot of Error Equations 52 and 53*
Unit errors resulting from angular misalignment about Y—Y axis as a function of weapon elevation for various cant angles, Cg



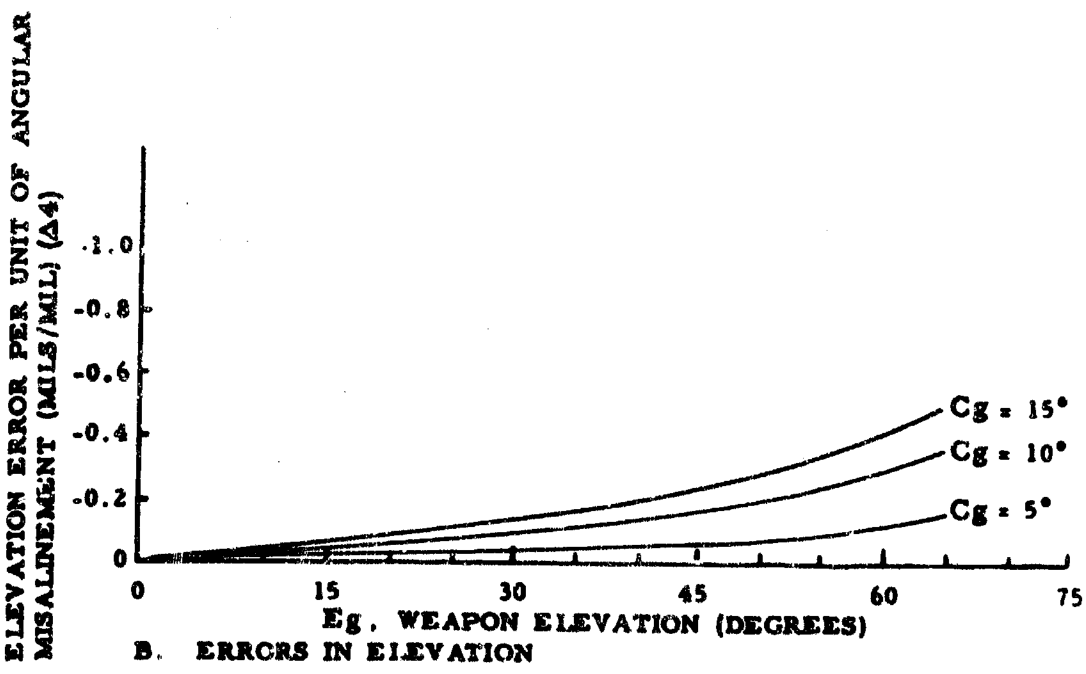
$$\frac{\text{Azimuth Error}}{\Delta 4} = \frac{\sin Es \cos Es \cos Cg}{1 - \sin^2 Es \cos^2 Cg} \qquad \text{(Eq. 54)}$$

$$\frac{\text{Elevation Error}}{\Delta 4} = -\frac{\sin Es \sin Cg}{\sqrt{1 - \sin^2 Es \cos^2 Cg}} \qquad \text{(Eq. 55)}$$

Figure 48. *Actuating arm elevating axis not parallel to trunnion axis (error in plane perpendicular to canted plane)*

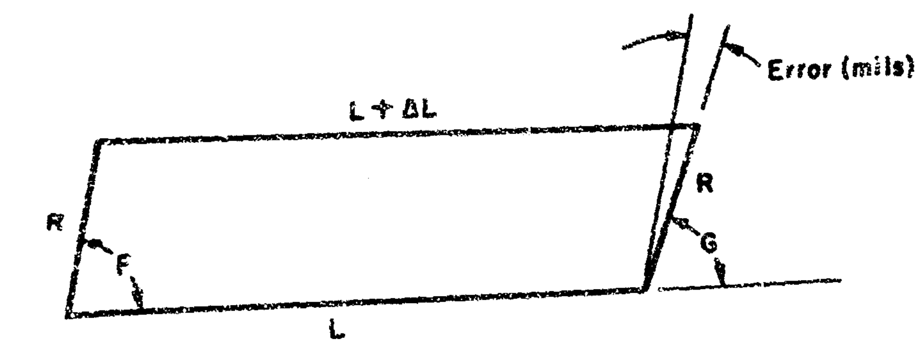


A. ERRORS IN AZIMUTH



B. ERRORS IN ELEVATION

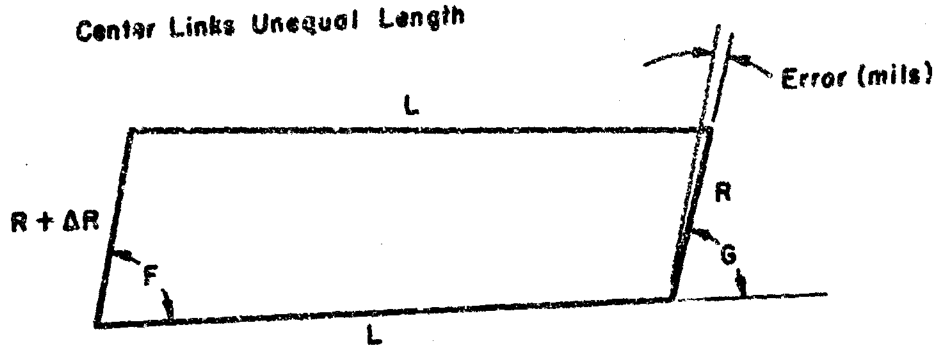
Figure 49. *Plot of Error Equations 54 and 55*
Unit errors resulting from angular misalignment about X—X axis as a function of weapon elevation for various cant angles, Cg



$F \neq G$

$$\text{Error (mils)} = -\left(\frac{\Delta L}{R} \times 10.00\right) \csc F \qquad \text{(Eq. 56)}$$

Center Links Unequal Length



$F \neq G$

$$\text{Error (mils)} = -\left(\frac{\Delta R}{R} \times 10.00\right) \cot F \qquad \text{(Eq. 57)}$$

Linkage Arms Unequal Length

Figure 50. *Errors in parallelogram linkages*

42. Standard Design Practices

The approach to the design of a compensating element has many facets. The instrument not only must be able to solve the compensation problem within the accuracy demanded by the system, but also must be able to perform under extreme conditions of field use, maintenance, and storage. In addition, the fabrication of the compensating element must be adaptable to standard manufacturing equipments and techniques. The designer of military equipment must take into consideration many factors that are not found in commercial designs. These factors are not always immediately evident to an individual new to the design of military equipment and often are the determining factors in fulfilling the ultimate requirements of the design of a compensating element. The following paragraphs give a discussion of these factors. Naturally, the weight given to each one will depend on the requirements of the specific problem at hand.

43. General Design Considerations

a. Raw Materials. The various properties of materials that determine their applicability to specific design problems are available to the designer in handbooks and vendors' catalogs. It is the purpose of these paragraphs, however, to call attention to factors that must be given special consideration in the selection of materials because of the military nature of the equipment being designed.

It must be assumed that fire control equipment will be exposed to every possible extreme in environmental conditions; from tropical to polar temperatures, from extreme dryness to immersion in salt water. and, in addition, to all forms of dirt and fungus. Corrosion is one of the most difficult problems that arise due to environment. Although this problem has never been completely solved, every attempt should be made to select materials that will minimize corrosion. If resistance to corrosion were the only factor requiring consideration, the problem would be less difficult. Usually a compromise must be reached that affords the best characteristics for a number of factors such as strength, weight, machinability, availability, etc. In reference to availability, the selection of material must be made on the basis of wartime availability. Many materials that would be considered optimum for specific parts and are readily available during peacetime, become critical because of stepped-up production requirements or scarce because of interruption of imports during wartime. When compromise results in other than optimum resistance to corrosion, then the item must be given the best possible protection by the use of suitable finishing processes.

Another cause of corrosion is the electrolytic action caused by the coupling of dissimilar materials. This condition is especially severe in the case of magnesium in combinations. Electrical isolation in junctions between dissimilar metals may be effected by the application of plastic coating materials.

In selecting dissimilar materials for assembly within the same instrument, the coefficients of expansion also must be taken into consideration. The environmental extremes to which military equipment is subjected often rule out particular materials because of excessive differences in their expansion coefficients. In other cases, it may be possible, by careful design of clearances, to accept the extremes in differential expansion.

b. Seals. A compensating element requires complete sealing to protect its interior from such products of adverse environment as dirt, dust, corrosion, moisture condensation, and fungus growth. Even normal field maintenance procedures on related system equipment are a potential source of damage to an incompletely sealed instrument. A typical example of this latter situation is the "mudding" procedure in which a water or steam jet is used to clean automotive equipment and tanks. These procedures can damage or make a compensating element unusable if it is not extremely well sealed, especially when optical instruments are employed. Therefore, the designer often must completely seal an instrument to satisfy the requirements of the weapon system in which it is used.

The design goal for all seals is the complete elimination of the process known as "breathing". Breathing is the expulsion of the contained atmosphere, or intake of the surrounding atmosphere, and tends to occur whenever there is a difference in pressure between the housing's internal atmosphere and the external atmosphere. It can be caused by changes in altitude, barometric pressure, or temperature. Since both expulsion and intake usually occur, introduction of moisture, dust, and fungus spores takes place, and in time an instrument may become useless. A seal that completely eliminates breathing is classified as a hermetic seal. True hermetic seals can be achieved in enclosures that do not include mechanical inputs or outputs. Such a design would involve only static sealing techniques. Dynamic seals at entry points for mechanical rotating parts present greater difficulty and, as yet, true hermetic seals for this purpose have not been developed.

In instrument housings containing both electrical or electronic and mechanical parts, sectionalizing techniques may be employed to achieve the maximum protection. All mechanical parts are placed in one section of the housing which is equipped with the necessary dynamic and static seals. The electrical and electronic equipment is placed in an adjacent section that is hermetically sealed. All electrical connections between sections and external equipment are made through hermetic connectors.

Good static seals for access covers can be obtained by keeping the area of contact at a minimum. Polytetra fluoroethylene coated parts or covers with polytetra fluoroethylene lips under screw or spring pressure can be used to obtain a seal. One method that has met with success employs a groove around the area to be sealed. After the cover is put into place, a liquid gasket material that remains viscous over the life of the equipment is injected into the groove. One disadvantage of this method is that bleeding of the gasket material may occur. However, the bleeding usually is so slight that it does not deteriorate the seal.

Breathing can occur through joints of a container or because of the porosity of the material from which the instrument enclosure is made. Joints must be welded with suitable materials to obtain a good hermetic seal. Several methods can be used to seal the pores of the material from which the instrument case is fabricated. The surface can be peened or impregnated. Peening can be accomplished by blasting with steel balls or other suitable materials. This procedure has the characteristic of altering the surface of the material by changing dimensions or hardness slightly, which in some cases may not be tolerable. Impregnating has been used with success in sealing castings. This type of treatment does not alter the surface of the material. When impregnating a housing or sealing the joints of optical instruments, use of varnishes or similar products should be avoided. These materials contain solvents that evaporate over a period of time and deposit a film on the glass thereby affecting its optical qualities.

To further the prevention of corrosion and moisture from forming in a heremetically sealed unit, sealed instruments can be filled with an inert gas such as nitrogen. This procedure does not eliminate the problem of breathing, however. It is quite common to make the pressure of the gas inside the container higher than the atmospheric pressure so that the breathing process will be outward and a dry atmosphere maintained within until pressure equalization takes place.

For dynamic seals required with rotating shafts or limited lateral movements of parts, reasonable success can be obtained by using synthetic rubber seal rings such as "O" rings. However, it usually is difficult to obtain a dynamic seal using "O" rings when diameters exceed 2 inches. If there is no rotation involved and only a limited lateral movement is required, flexible metallic bellows can be used to obtain a seal.

c. Finishes. Finishes are normally applied to provide protection against corrosion, to impart surface properties necessary to the function, and to alter the appearance of a part. Frequently, design requirements demand that a finish provide all these objectives. But since no single finish will provide all these properties, it must be decided which properties are more important, and which can be compromised.

Corrosion protection is the most frequently encountered finish problem because of the wide variety of environmental conditions to which military equipment may be exposed. When surface properties and appearance impose no restrictions, an inorganic (chemical or electrochemical) surface treatment with an organic finish (paint primer plus top coat) provides maximum protection against corrosion. When possible, organic coatings should be baked in preference to being air-dried. For the interior of optical instruments, organic thinners or carriers that tend to deposit a film on glass from their constant evaporation should be avoided when possible. Where a paint finish is impractical, such as on a working surface, electroplate or chemical surface treatment can be used to advantage but with an intermediate degree of corrosion protection. The coupling of dissimilar metals often produces corrosion over and above that which would normally occur in the metals separately. This type of coupling should be avoided if possible or the corrosive action eliminated by a finish.

Certain physical properties for a surface often are required to make a part function properly. Properties such as wear resistances, special friction charactristics, hardness, solderability, conductivity, and light absorption can be imparted by surface treatments. Most of these treatments also furnish corrosion protection, but none are as effective as the organic finish method. When tolerances are critical, the designer must take into consideration the amount of surface build-up contributed by a finish process.

The appearance of a finish for military equipment is always secondary. Special finishes are available for altering appearance, such as fast-drying camouflaging agents, that add little to protec against corrosion. Wrinkle enamels have been used to hide imperfections in base metals, but are difficult to decontaminate after exposure to atomic fallout. Mottle or "hammer" tone type enamel finishes are easier to clean and have imperfection hiding properties better than conventional enamels but not as good as wrinkle finishes.

In choosing a finish, the designer must be familiar with its final characteristics and the problems involved in processing. In addition, the designer must take into consideration the availability of the finish or the materials used in the finish during a period of emergency.

d. Bearings. Wherever motion is encountered in instrument design, the problem of friction must be coped with. Friction can result in rapid wear, decreased sensitivity, and inaccurate computation. In addition, friction must be kept to a minimum to insure proper operation at extreme temperatures. This even applies to manually operated parts or parts operated at low speeds.

Of the various bearing types available the antifriction type employing ball or roller elements is most frequently employed in fire control instrument design. Past experience in this type of design work has provided certain general principles in the use of antifriction bearings that the designer should recognize. Because of the low temperatures at which military equipment must be capable of operating, care must be exercised in tolerancing bearing fits to prevent failure or binding. Generally, ball bearing hole sizes should be such that the bearings can be installed by hand (push-fit) rather than by the force-fit method requiring a mechanical press. This practice not only prevents compression and shrinkage of bearing races with the attendant possibility of binding but also facilitates assembly and maintenance procedures. Also, special attention should be given to minimum end play tolerances so that differential expansion at temperature extremes cannot produce increased axial thrust loads and friction.

The increased demand for antifriction bearings during wartime may reduce the supply of special or close-tolerance types. Therefore, wherever possible, equipment should be designd with appropriate scale factors to give the required accuracy with standard or class B bearing types.

e. Gears. Gears are used to transmit or change direction of motion and to provide mechanical advantage, scale factors, and computations in mechanical or electromechanical systems. There are many types of gears and gear combinations. However, for reliability, and ease and speed in production, the designer should try to employ simple gear designs. Whenever possible, plain spur gears should be used in lieu of other types, since shaft end play then has minimum effect. Assembly procedures are simplified and the binding effects of differential expansion encountered at temperature extremes are reduced in importance. When bevel gears must be employed, avoid the use of the spiral tooth type. These gears afford extra smoothness but require careful end play adjustment and alignment because of their thrust reversal characteristics.

Torque values encountered in computing instruments are usually of a low enough magnitude that they do not complicate the gearing design. However, the mesh between gears can introduce backlash that might seriously affect the accuracy or calibration of an instrument. To obtain a minimum of backlash with maximum tolerances between shaft centers, gears are usually designed with 14.5-degree involute teeth. As a general rule of thumb, a 0.001-inch backlash tolerance per inch of pitch diameter, with a 0.001-inch tolerance on the pitch diameter itself, may be considered as acceptable. It is possible, is some cases, to choose a scale factor that will minimize further the error introduced by the backlash tolerance. Where backlash cannot be tolerated, split-type antibacklash gears or other antibacklash mechanisms can be utilized. If higher accuracies are desired, an adjustment should be provided to obtain proper gear mesh. Lapping should not be resorted to for obtaining accurate meshes, as this practice results in loose meshes after prolongd operation if the lapping compound has not been thoroughly removed from the gears. Even with the cleaning ability of sonic devices, the practice should still be avoided as the cleaning equipment may not be readily available to a manufacturer during an emergency.

44. Manufacture

During emergencies equipment may not be produced entirely by normal peacetime manufacturers of military equipment. More than likely, a good portion of equipment will be made by manufacturers who have converted or expanded their production to critical needs. For example, a manufacturer of household appliances might convert his production to the manufacture of a compensating element or parts thereof. In all probability his engineering staff has not had experience in the design or manufacture of fire control instruments, nor are his production facilities set up for this type of manufacture. Simplicity of design coupled with practical working tolerances can make the manufacture of a compensating element flexible enough to be handled by manufacturers not normally engaged in this type of production. These factors are discussed in greater details in the following paragraphs.

a. Flexibility. The design of a compensating element should be of a nature that readily lends itself to different types of manufacturing techniques. For example, if a part is to be made from a casting, it should be designed so it is immaterial which method of casting is used (investment, sand, permanent mold, etc.). To carry this concept further, it may be possible, in certain cases, to specify alternate forms of fabrication. A typical example is an instrument housing that is designed in a form that can be fabricated by casting methods, stamping and forming processes, drawing processes, or by welding separate pieces together. Whichever method is used, designs that require special tools or machines should be avoided. If this procedure is not followed, special machinery that is required for manufacture may not be available when needed or production may be limited because items will be obtainable only from one or a few manufacturers who have the special machinery and knowledge.

b. Tolerances. Correct tolerances often are the key to manufacturing success, especially when an item requires mass production. The problem of tolerances probably is one of the biggest factors in losing or wasting time and increasing manufacturing costs. This trouble can be caused by the existence of a gap between the tolerance a designer specifies for a dimension and the tolerance that can be achieved or held consistently in production. A designer should know the accuracy capablities of the types of machines that will be used in the fabrication of an item. During peacetime production, it may be known that certain manufacturers can obtain high accuracies consistently, so the tendency might exist to tolerance dimensions to fit the manufacturer. However, during an emergency different manufacturers may not be able to achieve the same accuracies because of the capabilities of their machines or lack of sufficiently trained personnel. Therefore, the tolerances used should be those that can be obtained with standard commercial tools and equipment. If it is shown that the instrument being designed will not satisfy the accuracy requirements using commercial tolerances, then an attempt should be made, before tightening tolerances, to improve the overall accuracy by increasing sizes or scale factors. In some cases, it may be necessary to take a different design approach. In general, the designer of military equipment should try to open all tolerances as much as possible within the limits of good manufacturing practices. As a rule of thumb, tolerances may be specified on the basis of 0.001-inch per inch wherever possible. Tighter tolerances are a necessity for items such as shaft centers (bearing holes), but even here center distances greater than 10 inches should not be held to less than 0.001-inch tolerance. A highly accurate jig is necessary to achieve such dimensions. Remember that the only information readily available to a manufacturer or Government inspector is located on the manufacturing drawings. Requests to open tolerances are time-consuming and many times result in serious delays in production.

c. Simplicity. Simplicity of design in military equipment lends itself to faster and easier production during a mobilization period. Every effort should be made to minimize the requirements for complex manufacturing processes and special machine tools and fixtures. Any additional design time spent in this effort can result in great savings of manufacturing time and reduce demands placed upon the limited supply of highly trained and skilled personnel.

Efforts to further design simplicity should be carried on throughout the fabrication of a prototype model. Here, production problems can be anticipated and eliminated by redesign instead of being solved at a later date by complicated fabrication processes.

d. Raw Material. Some of the factors concerning the selection of materials were discussed under General Design Considerations at the beginning of this section. It is the purpose here to stress the factors in material selection that affect the manufacture of an item. By thorough consideration of the possible supply and demand for instrument materials during wartime, it may be possible to avoid critical shortages. The aim should be to choose materials, whenever possible, whose demand will not exceed the production facilities of this country or whose supply will not be cut off because of importation restrictions during a war. To further facilitate material procurement during an emergency period, it may be possible to specify an alternate material or alternate grade of material. Also to be considered is the workability of materials. Where extensive machining or forming of any kind is called for and where the part is apt to require mass production, special attention should be given to selecting a material that will not hinder production because of fabrication difficulties.

45. Field Use

After the functional features of a fire control instrument have been designed, it must be put into a physical form that is adapted to field use. This involves environmental considerations and human engineering problems as discussed in the following paragraphs.

a. Operating Simplicity. Ideally, a compensating element should be capable of being operated properly by a person with limited training or slight familiarity with the equipment. To achieve such an ideal, operating controls must be kept as simple and as few in number as possible. This advantage can be appreciated further if the designer realizes that personnel other than the normal gun crew might be forced to operate a weapon. Under battle conditions, untrained personnel may be required to keep a weapon firing in the face of enemy attack. Also, the smaller number of operations or manipulations a trained gunner is required to perform before a round is fired increases speed and reduces chances for error.

There are many techniques that a designer can use in solving human engineering problems to simplify operation. Controls should be made simple and easy to operate and obvious as to their function, but should not be so easy to manipulate that they are liable to move if accidentally touched or brushed. The designer also must consider the readability of dials and leveling vials or other indicating devices, and location, position, and size of controls. Control knobs should be sufficiently large to be grasped easily, especially if the equipment will be used by personnel wearing gloves.

The phase of design that results in operating simplicity often does not receive the amount of attention by designers that it should. Many times, the key to obtaining maximum use and operating efficiency from equipment lies in facilitating its use by personnel.

b. Ruggedness. Military equipment must be rugged enough to withstand all the types of rough handling and mistreatment to which it might be subjected during transportation or field use. In general, military equipment receives more abuse than the nearest equivalent in commercial equipment. Vibration and shocks or jolts can place extraordinary stresses on a compensating element. The detrimental effects of these stresses can vary between loss of accuracy to actual physical damage or destruction. For example, worm and worm gear assemblies are often used in the gearing of computers or sight mounts. Where vibration exists, the usual type of worm and gear assembly may drift until it locks itself at about 5 degrees of rotation. When this amount of drift rotation affects accuracy, specially designed worm and gear assemblies that limit drift to 3 degrees may be employed. Another example of the detrimental effects of vibration is found when a weapon is fired or when it is hit or near missed by a projectile. Both these conditions can produce high frequency vibrations of high amplitude. Occasionally, the high frequency vibrations occur at a resonant frequency of parts, causing them to fail or shatter. In a typical case, an element in an optical sight shattered when the weapon on which it was mounted was fired. The high frequency vibrations originating in the weapon tube were of the exact frequency and amplitude to affect that particular element. A redesign that dampened the high frequency vibrations solved this problem. Another vibration problem arises from the method used in mounting equipment. This problem was presented during the development of a self-propelled weapon in which a compensating element was secured at one end only. At certain road speeds, the vibrations set up by the treads of the vehicle caused the equipment to break away from the mount.

One of the more common effects of vibration is the working loose of screws and bolts. Maximum insurance against vibration should be provided by the use of an appropriate locking device, of which many types are now available.

When only limited ruggedness can be designed into a portion of an instrument such as an electronic element, proper shock mounting can alleviate the effects of shock and vibration.

Another factor that must be considered in equipment design is the treatment it receives from personnel. Military equipment is frequently subject to rough handling, particularly during the excitement of military action. Also, it frequently occurs that personnel unintentionally abuse equipment during maintenance procedures, overstressing or damaging parts which are weaker than adjacent parts. Overstressed screws or bolts may not actually fail until subjected to normal shock and vibration at a later time. Specifying oversize fastenings or bolts and screws with special heads requiring special tools for tightening helps to counteract this type of abuse.

c. Reliability. Often reliability is interrelated with ruggedness. However, success in obtaining reliability is achieved through proper design and provision of a great enough safety factor for the materials and components that make up a compensating element. The exact reliabilty of new designs often is difficult to determine or accurately predict. Past experience with components, materials, and design techniques can give a designer an indication of the reliability he might expect and the safety factor he must provide. When past experience is not available, the designer must turn to environmental and accelerated life tests that can be performed in the laboratory for information on which to evaluate the reliability of a design. The laboratory test results then can be used as a basis for redesign of areas where insufficient reliability is indicated.

d. Transportability. To be of value for tactical operation, military equipment must be at the location where it is needed, when it is needed. The two factors that affect the degree of transportability are weight and size. Since most equipment is designed to be airborne if necessary or capable of being moved by personnel, a limit generally is placed on both weight and size. By the time weight and space have been allocated for such items as armor, engine, gun, radio equipment, etc., the compensating element designer may find that he has very stringent limits to meet. Weight and size then become very critical factors in design. Under these circumstances, lightweight materials and miniature components help solve many problems.

Equipment being designed for manual portability may require features that will enable it to be readily dismantled, portaged, and then quickly reassembled. The designer must consider the ease and manner of reassembly and alignment and adjustment procedures necessary to achieve readiness for operation.

c. Environment. Under field use, equipment is subjected to extremes in environmental conditions. The equipment on which a compensating element is mounted may be exposed to the elements for long periods of time. Under these conditions, temperature can vary to extremes, and instruments may be exposed to rain, fog, dew, snow, and dust or sand within a relatively short period of time. Another important consideration is immersion in or spray from salt water during embarkation or debarkation. Corrosion is one of the biggest problems arising from environmental conditions. In warmer and tropical climates the effects of high humidity and fungus growth often tend to become the prime detrimental factors. Some of the aspects of design for environment have been discussed previously under the headings of Raw Materials, Seals, and Finishes. By the proper selection of these items some of the detrimental effects of environment can be minimized. When it is not possible to satisfy low-temperature environmental requirements for a compensating element, it may be possible to provide accessory equipment so that proper and reliable operation can be obtained. These accessories may comprise strategically placed electrical heating elements inside an instrument housing, or electrically heated blankets and covers to protect the instrument.

46. Maintenance

The importance of being able to maintain military equipment in good operating condition is obvious. Maintainability therefore should be given prime consideration during the design of a new piece of equipment. Some of the more important design factors affecting ease of maintenance are discussed in the following paragraphs.

a. Accessibility. The key to maintenance accessibility is the proper location and packaging of components. The arrangement of components in a weapon system should afford ready access for maintenance purposes. When this is not possible because of space limitations, easy removal of the component for service should be insured by proper design and accessibility of mountings. Trouble-shooting and repair work on a component may be facilitated greatly by the provision of adequate and properly located access covers on all housings. In locating access covers the designer should attempt to foresee maintenance requirements. Expendable parts, such as fuses, electron tubes, illumination lamps, filters, etc., should be located for access without component removal.

b. Simplicity. During peacetime, trained personnel with experience are available for servicing equipment. However, during an emergency, the increased use of great quantities of complex equipment produces an acute need for additional maintenance personnel. The additional personnel are obtained from the ranks and many have had little or no experience in the instrument field. These people are given intensive short courses in maintenance, and without time to gain experience, become the "experts" who must maintain military equipment under wartime conditions that are usually other than optimum. Often, too, certain maintenance procedures must be performed in the field by the operating personnel who cannot be expected to have special talents in instrument work.

Hence, equipment should be straightforward in design and construction so that troubleshooting and maintenance can be performed from a common sense aspect. Tricks and gimmicks should be avoided in the design of equipment. Many times simplicity can be obtained by a better arrangement of parts. If a compensating element is inherently complex, so that simplification is difficult to obtain (as in a computer for an antiaircraft weapon system), checking circuits or monitoring devices can be built into the equipment. These features coupled with modular design, in which an instrument is divided into several, easily monitored, replaceable modules, can greatly simplify the task of inexperienced maintenance personnel in keeping equipment in an operating condition.

c. Standardization and Interchangeability. Both field and depot maintenance depend to a large degree on the availability of repair parts. The task of supplying and stocking such parts for the numerous types and large quantities of military equipment has attained enormous proportions. The only means of alleviating this situation is by standardization and designing for interchangeability. These two factors tend to reduce the number of different kinds of stock parts as well as simplifying replacement and installation procedures in maintenance work.

Standardization has two approaches. First, the designer should use previously standardized parts and subassemblies wherever possible, such as are found listed and specified in Federal stock lists. The second approach is that of standardizing subassemblies within a particular equipment. For example, if several amplifiers of similar function are required in a compensating element, it is often possible through proper design and selection to make a single amplifier design serve in all the functions. Or, in the case of servomotors and synchros, employing a single type and size throughout greatly furthers standardization. The standardization concept may be carried still further if items such as electronic or mechanical subassemblies of other current equipment are employed.

When a subassembly is to be standardized, the chief requirement that must be satisfied is interchangeabilty. Proper selection of scale factors and tolerances are essential. Adjustment points and fitting procedures to compensate for variations from standard require extra maintenance time and skill and should be kept to a minimum.

The extent to which a designer succeeds in standardization not only affects logistic problems and formal field and depot maintenance but also provides benefits that extend from manufacture to field use. The manufacturing process is simplified from the initial reduction in tooling and planning to the packaging and shipping of the finished product. In field use, standardization promotes familiarity with equipment and, in addition, makes possible the so-called cannibalization technique that often becomes important to field maintenance under emergency conditions.

d. Expendability. When it is shown that repairing a part, subassembly, or assembly is uneconomical both time and cost-wise in comparison to scrapping and replacing the item, then the item should be designed as an expendable unit. Some of the conditions under which items can be designed for expendability are: extremely accurate and permanent positioning of parts is required; extraordinary or complex assembly technique and equipment are required; or the major portion of a subassembly's cost lies in a part that is considered likely to wear and require replacement.

In designing an expendable item, permanent assembly techniques may be specified where practicable to simplify manufacture. Examples of these techniques are spinning, spot welding, riveting, and bonding.

47. Storage

A time lag always exists between the manufacture of equipment and actual field use. This time lag can become an extended period as when equipment is stockpiled for the future. Storage of equipment commences at the manufacturer's shipping point and extends to Government supply depots in this country and overseas. Storage facilities can vary from good warehouse protection to open field storage in forward areas that might result in direct exposure to the elements. In general, it may be assumed that equipment can be subjected to the same extremes in environmental conditions during storage as under actual field use. (See paragraph 45c.) Failure on the part of a designer to consider the possible effects of storage may result in equipment being unfit for use when it is needed.

Ideally, maintenance should not be necessary during storage. However, when parts are subjected to the effects of aging and environment such as corrosion, dust and vapor deposits due to breathing, or deterioration of lubricants, provision should be made to allow for periodic reconditioning. That is, the unit should be capable of being disassembled for cleaning and lubrication.

Many new types of packing procedures that improve preservation during storage are available in Government specifications. These include wax-dipping containers, canning, placing drying agents or moisture-absorbing agents within the containers, and charging containers with dry gas.

APPENDIX A
Plane Trigonometry

A brief review and summary of plane trigonometry relationship is included here for reference and as an aid in understanding the discussion of spherical trigonometry that follows.

a. Definitions.

(1) An angle may be considered as generated by a line which first coincides with one side (initial side, Figure A1) of the angle, then revolves about the vertex, and finally coincides with the other side (terminal side).

(2) An acute angle is less than 90 degrees.

(3) An obtuse angle is greater than 90 degrees and less than 180 degrees.

(4) A triangle is a right triangle if one of its angles is 90 degrees. If none of the angles are 90 degrees, it is referred to as an oblique triangle.

(5) An angle is said to lie in a certain quadrant (Figure A1) when its terminal side lies in that quadrant.

(6) The six trigonometric functions of an acute angle, such as A in Figure A2, are defined as follows:

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

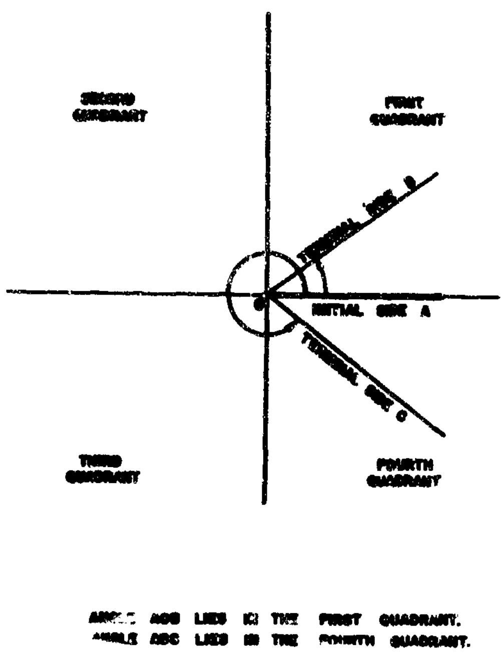


Figure A1. *The four quadrants*

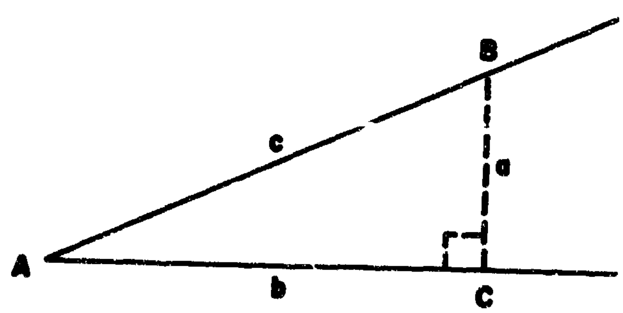


Figure A2. *Acute angle*

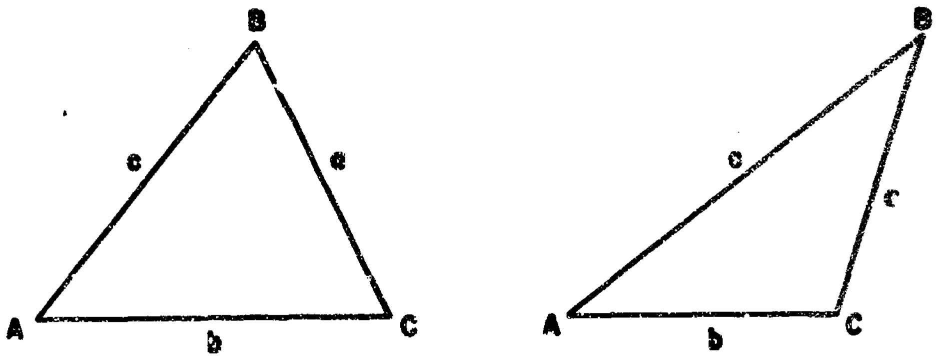


Figure A3. *Oblique triangles*

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{c}{a}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b}$$

$$\cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a}$$

(7) The numerical values of angular functions increase or decrease as tabulated below when the angles increase as shown in the column headings.

Function	In Quad. I 0 to 90°	In Quad. II 90 to 180°	In Quad. III 180 to 270°	In Quad. IV 270 to 360°
sine	0 to +1	+1 to 0	0 to −1	−1 to 0
cosine	+1 to 0	0 to −1	−1 to 0	0 to +1
tangent	0 to +∞	−∞ to 0	0 to + ∞	−∞ to 0
cosecant	+∞ to +1	+1 to +∞	−∞ to −1	−1 to −∞
secant	+1 to +∞	−∞ to −1	−1 to −∞	+∞ to +1
cotangen	+∞ to 0	0 to −∞	+∞ to 0	0 to −∞

b. Trigonometric Relationships. The six trigonometric functions of an angle A satisfy the relations:

$$(1)\ \sin A = \frac{\cos A}{\cot A} = \frac{1}{\csc A} = \cos A \tan A = \frac{\tan A}{\sec A} = \sqrt{1 - \cos^2 A}$$

$$(2)\ \cos A = \frac{\sin A}{\tan A} = \frac{1}{\sec A} = \sin A \cot A = \frac{\cot A}{\csc A} = \sqrt{1 - \sin^2 A}$$

$$(3)\ \tan A = \frac{\sin A}{\cos A} = \frac{1}{\cot A} = \sin A \sec A = \frac{\sec A}{\csc A} = \sqrt{\sec^2 A - 1}$$

$$(4)\ \csc A = \frac{\cot A}{\cos A} = \frac{1}{\sin A} = \sec A \cot A = \frac{\sec A}{\tan A} = \sqrt{1 + \cot^2 A}$$

$$(5)\ \sec A = \frac{\tan A}{\sin A} = \frac{1}{\cos A} = \tan A \csc A = \frac{\csc A}{\cot A} = \sqrt{1 + \tan^2 A}$$

$$(6)\ \cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A} = \cos A \csc A = \frac{\csc A}{\sec A} = \sqrt{\csc^2 A - 1}$$

c. Reduction of Trigonometric Functions to Functions of Acute Angles.

(1) Reduction of formulas for angles lying in the second quadrant are:

$\sin (180° - A) = \sin A$

$\cos (180° - A) = -\cos A$

$\tan (180° - A) = -\tan A$

$\csc (180° - A) = \csc A$

$\sec (180° - A) = -\sec A$

$\cot (180° - A) = -\cot A$

(2) Reduction formulas for angles lying in the third quadrant are:

$\sin (180° + A) = -\sin A$

$\cos (180° + A) = -\cos A$

$\tan (180° + A) = \tan A$

$\csc (180° + A) = -\csc A$

$\sec (180° + A) = -\sec A$

$\cot (180° + A) = \cot A$

(3) Reduction formulas for angles lying in the fourth quadrant are:

$\sin (360° - A) = -\sin A$

$\cos (360° - A) = \cos A$

$\tan (360° - A) = -\tan A$

$\csc (360° - A) = -\csc A$

$\sec (360° - A) = \sec A$

$\cot (360° - A) = -\cot A$

d. The Oblique Triangle. The trigonometric solution of oblique triangles depends upon the application of three laws. See Figure A3.

(1) Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ (Eq. A1)}$$

(2) Law of Cosines

(a) For Sides

$a^2 = b^2 + c^2 - 2bc \cos A$ (Eq. A2)

$b^2 = a^2 + c^2 - 2ac \cos B$ (Eq. A3)

$c^2 = a^2 + b^2 - 2ab \cos C$ (Eq. A4)

(b) For Angles

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \text{(Eq. A5)}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \qquad \text{(Eq. A6)}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \qquad \text{(Eq. A7)}$$

(3) Law of Tangents

$$\frac{a + b}{a - b} = \frac{\tan \tfrac{1}{2}(A + B)}{\tan \tfrac{1}{2}(A - B)} \qquad \text{(Eq. A8)}$$

$$\frac{c + a}{c - a} = \frac{\tan \tfrac{1}{2}(C + A)}{\tan \tfrac{1}{2}(C - A)} \qquad \text{(Eq. A9)}$$

$$\frac{b + c}{b - c} = \frac{\tan \tfrac{1}{2}(B + C)}{\tan \tfrac{1}{2}(B - C)} \qquad \text{(Eq. A10)}$$

e. Other Useful Trigonometric Identities.

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A + \tan B}{1 \mp \tan A \tan B}$$

$$\cot (A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

$$\cos B - \cos A = 2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

$$\tan A + \tan B = \frac{\sin (A + B)}{\cos A \cos B}$$

$$\tan A - \tan B = \frac{\sin (A - B)}{\cos A \cos B}$$

$$\cot A + \cot B = \frac{\sin (B + A)}{\sin A \sin B}$$

$$\cot A - \cot B = \frac{\sin (B - A)}{\sin A \sin B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A}$$

$$\sin \tfrac{1}{2}A = \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \frac{\sin A}{1 + \cos A}$$

$$\cot \tfrac{1}{2}A = \frac{\sin A}{1 - \cos A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$$

$$\cot^2 A = \frac{1 + \cos 2A}{1 - \cos 2A}$$

$$\sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B)$$

$$\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$$

$$\frac{\sin A \pm \sin B}{\cos A + \cos B} = \tan \tfrac{1}{2} (A \pm B)$$

$$\frac{\sin A \pm \sin B}{\cos B - \cos A} = \cot \tfrac{1}{2} (A \mp B)$$

APPENDIX B

Basic Spherical Trigonometry

Since basic spherical trigonometry is not universally taught, the fundamental relationships in this subject are covered below for those readers who are not familiar with the subject. Readers who are familiar with the laws of spherical trigonometry may utilize this information as a reference and review.

a. Definitions.

(1) A great circle (Figure B1) is a circle that is formed by the intersection of a plane through the center of a sphere and the sphere.

(2) A spherical triangle (Figure B1) consists of three arcs of great circles that form the boundaries of a portion of a spherical surface. The vertices of spherical triangles are denoted by A, B, and C, and the opposite sides by a, b, and c as in plane trigonometry.

(3) The magnitude of an angle of a spherical triangle is that of the plane angle formed by the tangents to the sides of the angle at its vertex, or that of the dihedral angle between the planes of the great circles forming the angle.

(4) The planes of the great circles bounding a spherical triangle form a trihedral angle at the center of the sphere. The face angles of this trihedral angle, being measured by their intercepted arcs, are designated by the same letters as the corresponding sides of the spherical triangle.

(5) A right spherical triangle is a spherical triangle one of whose angles is 90 degrees. An oblique spherical triangle is one, none of whose angles is 90 degrees.

b. The Right Spherical Triangle.

(1) Figure B2 will be used to find the relationships between the sides and angles of a right spherical triangle. In this illustration, arcs AB, BC, and AC form a spherical triangle on the surface of a sphere whose radius is unity. Angle C of this spherical triangle is 90 degrees so triangle ABC is a right spherical triangle.

(2) Consider a plane through point A (Figure B2) perpendicular to radius OB. From this figure it can be seen that:

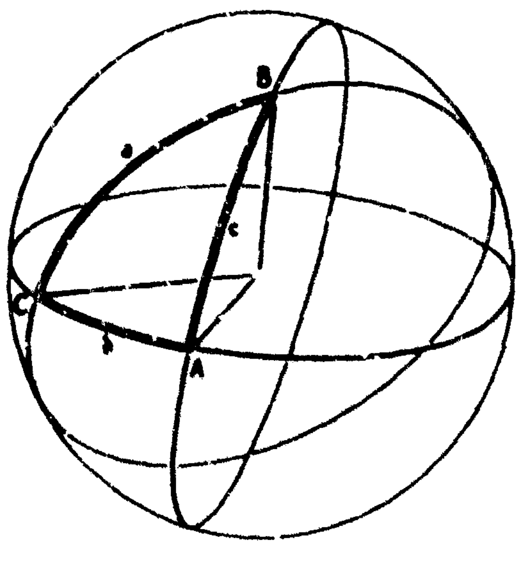


Figure B1. *Spherical triangle*

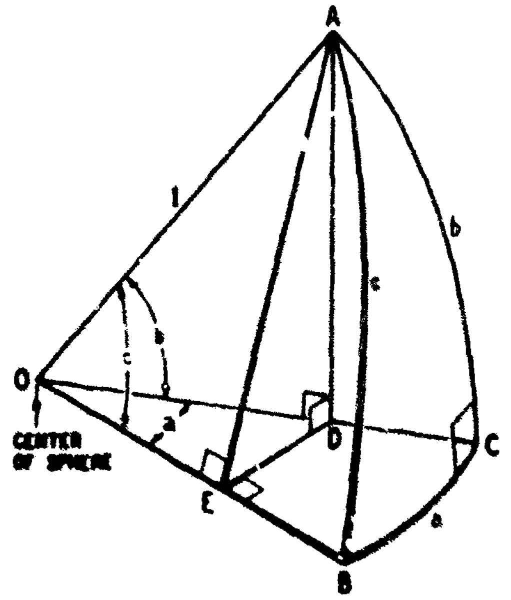


Figure B2. *Right spherical triangle*

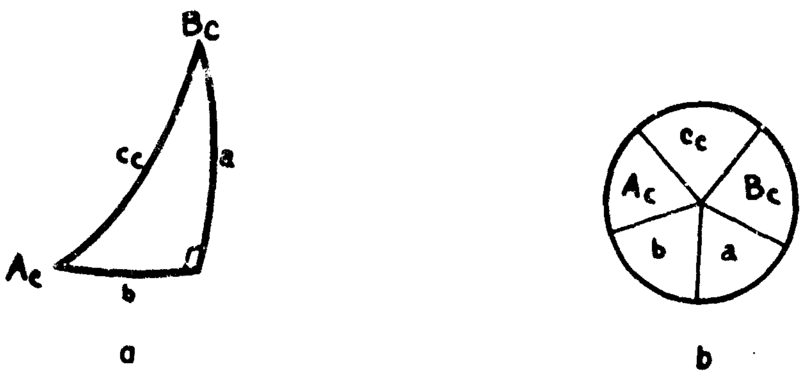


Figure B3. *Triangle and notation for Napier's rules*

$$OD = \cos b$$

$$OE = OD \cos a = \cos a \cos b$$

$$OE = \cos c$$

$$\cos c = \cos a \cos b \qquad \text{(Eq. B1)}$$

$$ED = OD \sin a$$

$$AD = \sin b$$

$$\tan E = \frac{AD}{ED} = \frac{\sin b}{\cos b \sin a} = \frac{\tan b}{\sin a}$$

$$\angle E = \angle B$$

$$\tan B = \frac{\tan b}{\sin a}$$

$$\sin a = \frac{\tan b}{\tan B}$$

$$\sin a = \tan b \cot B \qquad \text{(Eq. B2)}$$

$$AE = \sin c$$

$$\sin E = \frac{AD}{AE} = \frac{\sin b}{\sin c}$$

$$\sin B = \frac{\sin b}{\sin c}$$

$$\sin b = \sin B \sin c \qquad \text{(Eq. B3)}$$

$$ED = OE \tan a = \cos c \tan a$$

$$\cos E = \frac{ED}{AE} = \frac{\cos c \tan a}{\sin c} = \cot c \tan a$$

$$\cos B = \cot c \tan a \qquad \text{(Eq. B4)}$$

(3) In a similar manner, six additional basic relationships for the right spherical triangle can be developed. These are:

$$\sin a = \sin c \sin A \qquad \text{(Eq. B5)}$$

$$\cos A = \cos a \sin B \qquad \text{(Eq. B6)}$$

$$\cos B = \cos b \sin A \qquad \text{(Eq. B7)}$$

$$\cos A = \tan b \cot c \qquad \text{(Eq. B8)}$$

$$\sin b = \tan a \cot A \qquad \text{(Eq. B9)}$$

$$\cos c = \cot A \cot B \qquad \text{(Eq. B10)}$$

c. Napier's Rules of Circular Parts.

(1) The relationships B1 through B10 may be shown to follow from two very useful rules discovered by Baron Napier. For this purpose, the two legs, the complements of the two angles, and the complement of the hypotenuse are called the five circular parts of the right spherical triangle (Figure B3).

(2) If the five parts of this triangle are written down in their proper order as on the circumference of a circle, as shown in Figure B3 part b, Napier's rules may be applied. Any one of these parts may be called a middle part; then the two parts immediately adjacent to it are called adjacent parts, and the other two, opposite parts. For example, if a is taken as a middle part, B_c and b are the adjacent parts while c_c and A_c are the opposite parts.

Rule 1. The sine of any middle part is equal to the product of the tangents of the adjacent parts.

Rule 2. The sine of any middle part is equal to the product of the cosines of the opposite parts.

Note: In the employment of Napier's rules, the complement must be used for certain angles and sides. (See Figure B3.) To simplify notation, the subscript indicates that the complement is to be used. For example, A_c is $(90 - A)$, c_c is $(90 - c)$, and B_c is $(90 - B)$.

(3) Napier's rules may be verified by applying them in turn to each one of the five circular parts taken as a middle part and comparing the results with Equations B1 through B10. For example, let c_c, be taken as a middle part; then A_c and B_c are the adjacent parts, while a and b are the opposite parts:

Then, by Rule 1,

$$\sin c_c = \tan A_c \tan B_c$$

$$\sin(90 - c) = \tan (90 - A) \tan (90 - B)$$

or,

$$\cos c = \cot A \cot B$$

which agrees with Equation B10.

By Rule 2,

$$\sin c_c = \cos a \cos b$$

$$\sin (90 - c) = \cos a \cos b$$

or,

$$\cos c = \cos a \cos b$$

which agrees with Equation B1.

d. The Oblique Spherical Triangle.

(1) The Law of Sines. In the oblique spherical triangle ABC (Figure B4), h is drawn from A perpendicular to the side BC. This divides the oblique triangle into two right spherical triangles.

Applying Equation B3 to triangle ABD (Figure B4) gives:

$$\sin h = \sin B \sin c$$

Similarly, from triangle ACD and Equation B3:

$$\sin h = \sin C \sin b$$

Equating the above values of $\sin h$ gives:

$$\sin B \sin c = \sin C \sin b$$

$$\frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

In a like manner this expression can be extended to include angle A and side a, giving the following expression which is the Law of Sines:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad \text{(Eq. B11)}$$

(2) The Law of Cosines of Sides. In triangles ABD and ACD (Figure B4), applying Equation B1 gives:

$$\cos c = \cos BD \cos h$$

$$\cos b = \cos (a - BD) \cos h$$

$$\frac{\cos b}{\cos c} = \frac{\cos (a - BD)}{\cos BD}$$

$$\frac{\cos b}{\cos c} = \frac{\cos a \cos BD + \sin a \sin BD}{\cos BD}$$

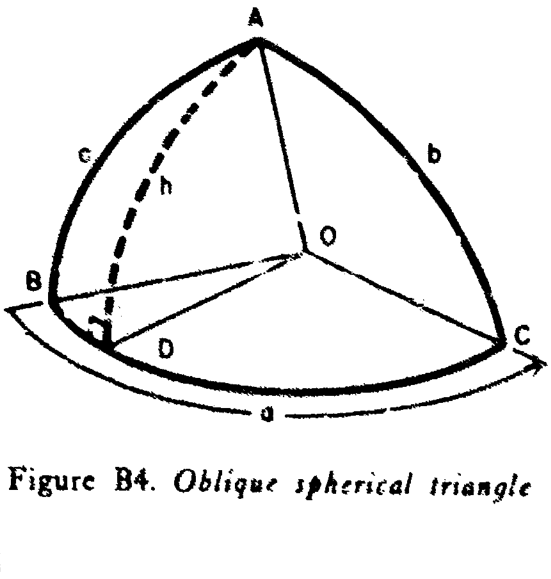


Figure B4. *Oblique spherical triangle*

$$\cos b = \cos c \cos a + \cos c \sin a \tan BD$$

but:

$$\cos B = \tan BD \cot c \text{ (See Equation B4)}$$

$$\tan BD = \frac{\cos B}{\cot c}$$

$$\tan BD = \cos B \tan c$$

so:

$$\cos b = \cos c \cos a + \cos c \sin a \cos B \tan c$$

$$\cos b = \cos c \cos a + \sin c \sin a \cos B \qquad \text{(Eq. B12)}$$

Equation B12 expresses the Law of Cosines of Sides. Similar expressions may be derived for $\cos a$ and $\cos c$.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \qquad \text{(Eq. B13)}$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C \qquad \text{(Eq. B14)}$$

(3) The Law of Cosines of Angles. In like manner, the following relations may be derived for the angles of triangle ABC (Figure B4):

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a \qquad \text{(Eq. B15)}$$

$$\cos B = -\cos A \cos C + \sin A \sin C \cos b \qquad \text{(Eq. B16)}$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c \qquad \text{(Eq. B17)}$$

(4) Auxiliary Formulas for the Solution of Oblique Spherical Triangles (Figure B4).

Problem 1. Given, three sides. The equations for finding the three angles are:

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}} \qquad \text{(Eq. B18)}$$

$$\tan \tfrac{1}{2} B = \sqrt{\frac{\sin (s-a) \sin (s-c)}{\sin s \sin (s-b)}} \qquad \text{(Eq. B19)}$$

$$\tan \tfrac{1}{2} C = \sqrt{\frac{\sin (s-a) \sin (s-b)}{\sin s \sin (s-c)}} \qquad \text{(Eq. B20)}$$

where

$$s = \tfrac{1}{2} (a+b+c)$$

$$s-a = \tfrac{1}{2} (b+c-a)$$

$$s-b = \tfrac{1}{2} (a-b+c)$$

$$s-c = \tfrac{1}{2} (a+b-c)$$

Problem 2. Given, three angles. The equations for finding the three sides are:

$$\tan \tfrac{1}{2} a = \sqrt{\frac{-\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)}} \qquad \text{(Eq. B21)}$$

$$\tan \tfrac{1}{2} b = \sqrt{\frac{-\cos S \cos (S-B)}{\cos (S-A) \cos (S-C)}} \qquad \text{(Eq. B22)}$$

$$\tan \tfrac{1}{2} c = \sqrt{\frac{-\cos S \cos (S-C)}{\cos (S-A) \cos (S-B)}} \qquad \text{(Eq. B23)}$$

where:

$$S = \tfrac{1}{2} (A+B+C)$$

$$S-A = \tfrac{1}{2} (B+C-A)$$

$$S-B = \tfrac{1}{2} (A-B+C)$$

$$S-C = \tfrac{1}{2} (A+B-C)$$

Problem 3. Given, two sides and the included angle. The equations for finding the other angles and side are:

$$\tan \tfrac{1}{2} (A+B) = \frac{\cos \tfrac{1}{2} (a-b)}{\cos \tfrac{1}{2} (a+b)} \cot \tfrac{1}{2} C \qquad \text{(Eq. B24)}$$

$$\tan \tfrac{1}{2} (A-B) = \frac{\sin \tfrac{1}{2} (a-b)}{\sin \tfrac{1}{2} (a+b)} \cot \tfrac{1}{2} C \qquad \text{(Eq. B25)}$$

$$\cos \tfrac{1}{2} C = \frac{\cos \tfrac{1}{2} (a+b)}{\cos \tfrac{1}{2} (A+B)} \sin \tfrac{1}{2} C \qquad \text{(Eq. B26)}$$

Problem 4. Given, two angles and the included side. The equations for finding the other sides and angle are:

$$\tan \tfrac{1}{2}(a+b) = \frac{\cos \tfrac{1}{2}(A-B)}{\cos \tfrac{1}{2}(A+B)} \tan \tfrac{1}{2} c \qquad \text{(Eq. B27)}$$

$$\tan \tfrac{1}{2}(a-b) = \frac{\sin \tfrac{1}{2}(A-B)}{\sin \tfrac{1}{2}(A+B)} \tan \tfrac{1}{2} c \qquad \text{(Eq. B28)}$$

$$\cos \tfrac{1}{2} C = \frac{\sin \tfrac{1}{2}(A+B)}{\cos \tfrac{1}{2}(a-b)} \cos \tfrac{1}{2} c \qquad \text{(Eq. B29)}$$

Problem 5. Given, two sides and an angle opposite one of them. The equations for finding the remaining parts, in certain cases, produce more than one solution. However, in practical use, the shape of the problem triangle is generally known beforehand and the applicable solution can be identified by inspection.

$$\sin B = \frac{\sin A \sin b}{\sin a} \qquad \text{(Eq. B30)}$$

$$\tan \tfrac{1}{2} c = \frac{\sin \tfrac{1}{2}(A+B)}{\sin \tfrac{1}{2}(A-B)} \tan \tfrac{1}{2}(a-b) \qquad \text{(Eq. B31)}$$

$$\cot \tfrac{1}{2} C = \frac{\sin \tfrac{1}{2}(a+b)}{\sin \tfrac{1}{2}(a-b)} \tan \tfrac{1}{2}(A-B) \qquad \text{(Eq. B32)}$$

Problem 6. Given, two angles and a side opposite one of them. The statement made in Problem 5 pertaining to multiple solutions applies to Problem 6 also.

$$\sin B = \frac{\sin A \sin b}{\sin a} \qquad \text{(Eq. B33)}$$

$$\tan \tfrac{1}{2} c = \frac{\sin \tfrac{1}{2}(A+B)}{\sin \tfrac{1}{2}(A-B)} \tan \tfrac{1}{2}(a-b) \qquad \text{(Eq. B34)}$$

$$\cot \tfrac{1}{2} C = \frac{\sin \tfrac{1}{2}(a+b)}{\sin \tfrac{1}{2}(a-b)} \tan \tfrac{1}{2}(A-B) \qquad \text{(Eq. B35)}$$

APPENDIX C

Derivation of Compensation Equations

a. Equations 15 and 16. Equations 15 and 16 as presented in paragraph 15 (Section III) were derived by means of coordinate rotation and matrix algebra as follows: Referring to Figure C1, the pitch and cant of the mount (Lm and Cm) and the relative traverse (Td) are known. It is desired to determine equations which can be solved for pitch and cant of the weapon (Lg and Cg). Rotation of the coordinate reference frames will be considered plus (+) in the direction shown in the sketch at the bottom of Figure C1.

(1) The vertical axis $X3$ is selected as the starting point and is given the value of unity. This vector has no components in the horizontal plane. Its coordinates, therefore, are:

$$X1^0 = 0$$

$$X2^0 = 0$$

$$X3^0 = 1$$

(2) Rotating the frame of these coordinates through angle Cm about the $X1^0$ axis, the new coordinates formed are derived from the matrix array for $-\Phi$ rotation:

$$\left.\begin{matrix} X1^1 \\ X2^1 \\ X3^1 \end{matrix}\right\} = \left\{\begin{matrix} 1 & 0 & 0 \\ 0 & \cos Cm & -\sin Cm \\ 0 & \sin Cm & \cos Cm \end{matrix}\right\}\left\{\begin{matrix} X1^0 \\ X2^0 \\ X3^0 \end{matrix}\right\}$$

$$X1^1 = X1^0 = 1 \times 0 + 0 \times 0 + 0 \times 1 = 0$$

$$X2^1 = 0 \times 0 + \cos Cm \times 0 - \sin Cm \times 1 = -\sin Cm$$

$$X3^1 = 0 \times 0 + \sin Cm \times 0 + \cos Cm \times 1 = \cos Cm$$

(3) Rotating through the angle Lm about the $X2^1$ axis, the new coordinates formed are derived from the array for $+\psi$ rotation:

$$\left.\begin{matrix} X1^2 \\ X2^2 \\ X3^2 \end{matrix}\right\} = \left\{\begin{matrix} \cos Lm & 0 & -\sin Lm \\ 0 & 1 & 0 \\ \sin Lm & 0 & \cos Lm \end{matrix}\right\}\left\{\begin{matrix} X1^1 \\ X2^1 \\ X3^1 \end{matrix}\right\}$$

$$X1^2 = \cos Lm \times 0 + 0 \times -\sin Cm - \sin Lm \cos Cm$$

$$X2^2 = X2^1 = 0 \times 0 + 1 \times -\sin Cm + 0 \times \cos Cm = -\sin Cm$$

$$X3^2 = \sin Lm \times 0 + 0 \times -\sin Cm + \cos Lm \cos Cm = \cos Lm \cos Cm$$

(4) Rotating through the angle Td about the $X3^2$ axis, the new coordinates are derived from the array for $-\theta$ rotation (intermediate operations omitted):

$$\left.\begin{matrix} X1^3 \\ X2^3 \\ X3^3 \end{matrix}\right\} = \left\{\begin{matrix} \cos Td & -\sin Td & 0 \\ \sin Td & \cos Td & 0 \\ 0 & 0 & 1 \end{matrix}\right\}\left\{\begin{matrix} X1^2 \\ X2^2 \\ X3^2 \end{matrix}\right\}$$

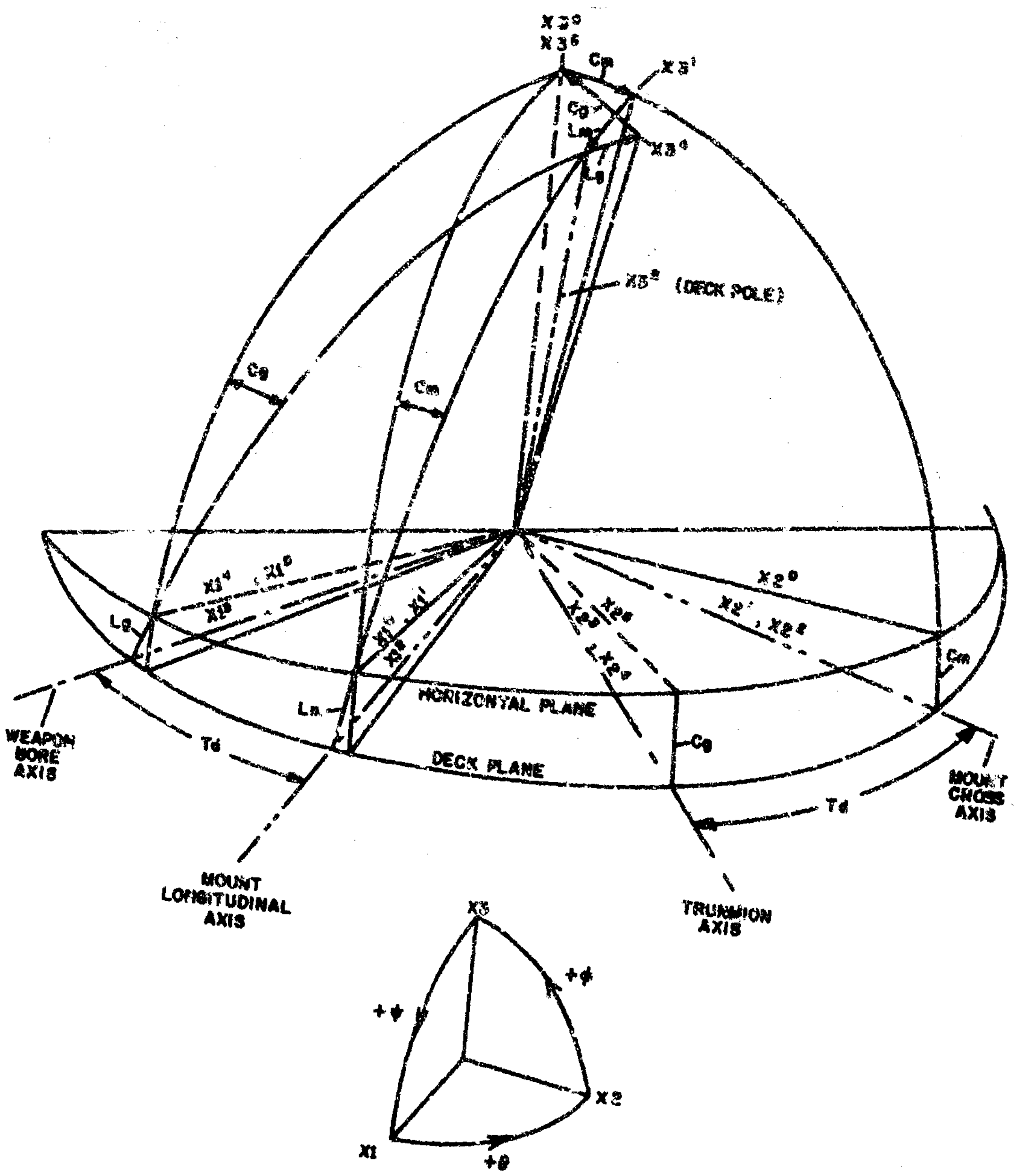


Figure C1. *Geometry for derivation of Equations 15 and 16*

$$X1^3 = -\cos Td \sin Lm \cos Cm + \sin Td \sin Cm$$

$$X2^3 = -\sin Td \sin Lm \cos Cm - \cos Td \sin Cm$$

$$X3^3 = X3^2 = \cos Lm \cos Cm$$

(5) Rotating through the angle Lg about the $X2^3$ axis until the $X1^4$ component is equal to zero, the new coordinates are derived from the array for $-\psi$ rotation:

$$\begin{Bmatrix} X1^4 \\ X2^4 \\ X3^4 \end{Bmatrix} = \begin{Bmatrix} \cos Lg & 0 & \sin Lg \\ 0 & 1 & 0 \\ -\sin Lg & 0 & \cos Lg \end{Bmatrix} \begin{Bmatrix} X1^3 \\ X2^3 \\ X3^3 \end{Bmatrix}$$

$$\begin{aligned} X1^4 = {} & \cos Lg\ (-\cos Td \sin Lm \cos Cm + \sin Td \sin Cm) \\ & + \sin Lg \cos Lm \cos Cm \end{aligned}$$

$$X2^4 = X2^3 = -\sin Td \sin Lm \cos Cm - \cos Td \sin Cm$$

$$\begin{aligned} X3^4 = {} & -\sin Lg\ (-\cos Td \sin Lm \cos Cm + \sin Td \sin Cm) \\ & + \cos Lg \cos Lm \cos Cm \end{aligned}$$

(6) Rotating through the angle Cg about the $X1^4$ axis until the $X2^5$ component is equal to zero, the new coordinates are derived from the array for $+\phi$ rotation:

$$\begin{Bmatrix} X1^5 \\ X2^5 \\ X3^5 \end{Bmatrix} = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos Cg & \sin Cg \\ 0 & -\sin Cg & \cos Cg \end{Bmatrix} \begin{Bmatrix} X1^4 \\ X2^4 \\ X3^4 \end{Bmatrix}$$

$$X1^5 = \cos Lg\ (-\cos Td \sin Lm \cos Cm + \sin Td \sin Cm) + \sin Lg \cos Lm \cos Cm = 0$$
(see Eq. 15)

$$\begin{aligned} X2^5 = {} & \cos Cg\ (-\sin Td \sin Lm \cos Cm - \cos Td \sin Cm) + \sin Cg\ [\ -\sin Lg\ (-\cos Td \\ & \sin Lm \cos Cm + \sin Td \sin Cm) + \cos Lg \cos Lm \cos Cm] = 0 \end{aligned}$$
(see Eq. 16)

Also:

$$\begin{aligned} X3^5 = X3^9 = {} & -\sin Cg\ (-\sin Td \sin Lm \cos Cm - \cos Td \sin Cm) + \cos Cg\ [\ -\sin Lg \\ & (\cos Td \sin Lm \cos Cm + \sin Td \sin Cm) + \cos Lg \cos Lm \cos Cm\ ] = 1 \end{aligned}$$

b. Equations 20 and 21.

From Figure C2:

$$y0 = y1 \cos Dt + z1 \sin Dt$$

$$z0 = z1 \cos Dt - y1 \sin Dt$$

$$x0 = x1$$

$$\tan At = \frac{x0}{y0} = \frac{x1}{y1 \cos Dt + z1 \sin Dt}$$

But from Figure 23 (Section III):

$$z1 = \sin Eg$$

$$y1 = \cos Eg \cos Tt$$

$$x1 = \cos Eg \sin Tt$$

$$\tan At = \frac{\cos Eg \sin Tt}{\cos Eg \cos Tt \cos Dt + \sin Eg \sin Dt}$$

Dividing through by cos Eg:

$$\tan At = \frac{\sin Tt}{\cos Tt \cos Dt + \tan Eg \sin Dt} \qquad \text{(see E 20)}$$

$$\sin Ea = z0 = z1 \cos Dt - y1 \sin Dt$$

$$\sin Ea = \sin Eg \cos Dt - \cos Eg \cos Tt \sin Dt \qquad \text{(see Eq. 21)}$$

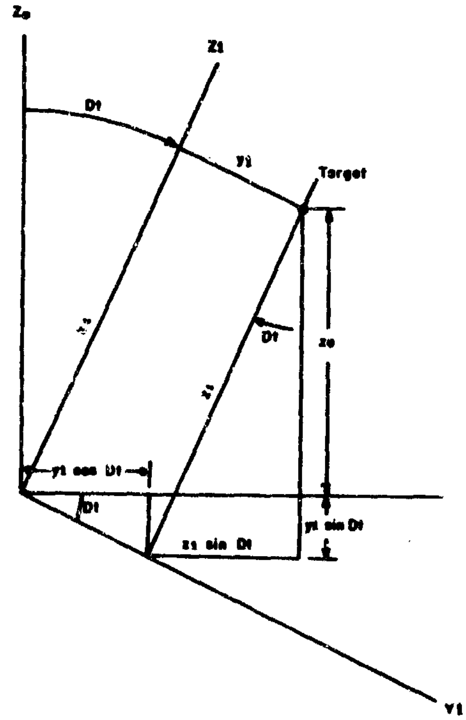


Figure C2. *Geometry for derivation of Equations 20 and 21*

c. Equations 45, 46, and 47 (Section V).

See Figure C3.

From triangle ADO:

$$\overline{AD} = R \sin Ea \tag{Eq. C1}$$

and from triangle ADB:

$$\overline{AD} = \overline{AB} \sin E \tag{Eq. C2}$$

Let $R = 1$, and combine Equation C1 with Equation C2:

$$\sin Ea = \overline{AB} \sin E \tag{Eq. C3}$$

but from right triangle OAB:

$$\overline{AB}^2 = 1 - \overline{OB}^2 \tag{Eq. C4}$$

and from right triangle OBC:

$$\overline{OB} = \sin \Delta 1 \tag{Eq. C5}$$

Substituting Equation C5 in Equation C4:

$$\overline{AB}^2 = 1 - \sin^2 \Delta 1 = \cos^2 \Delta 1$$

or

$$\overline{AB} = \cos \Delta 1 \tag{Eq. C6}$$

Therefore, substituting Equation C6 in Equation C3:

$$\sin Ea = \cos \Delta 1 \sin E \tag{Eq. C7}$$

and Equation C7 is identical to Equation 45.

From right triangle ODB:

$$\tan \delta 1 = \frac{\overline{OB}}{\overline{BD}} \tag{Eq. C8}$$

and from right triangle ABD:

$$\overline{BD} = \overline{AB} \cos E \tag{Eq. C9}$$

substituting Equation C6 in Equation C9:

$$\overline{BD} = \cos \Delta 1 \cos E \tag{Eq. C10}$$

Therefore, substituting Equations C5 and C10 in Equation C8:

$$\tan \delta 1 = \frac{\sin \Delta 1}{\cos \Delta 1 \cos E} = \frac{\tan \Delta 1}{\cos E} \tag{Eq. C11}$$

and Equation C11 is identical to Equation 46.

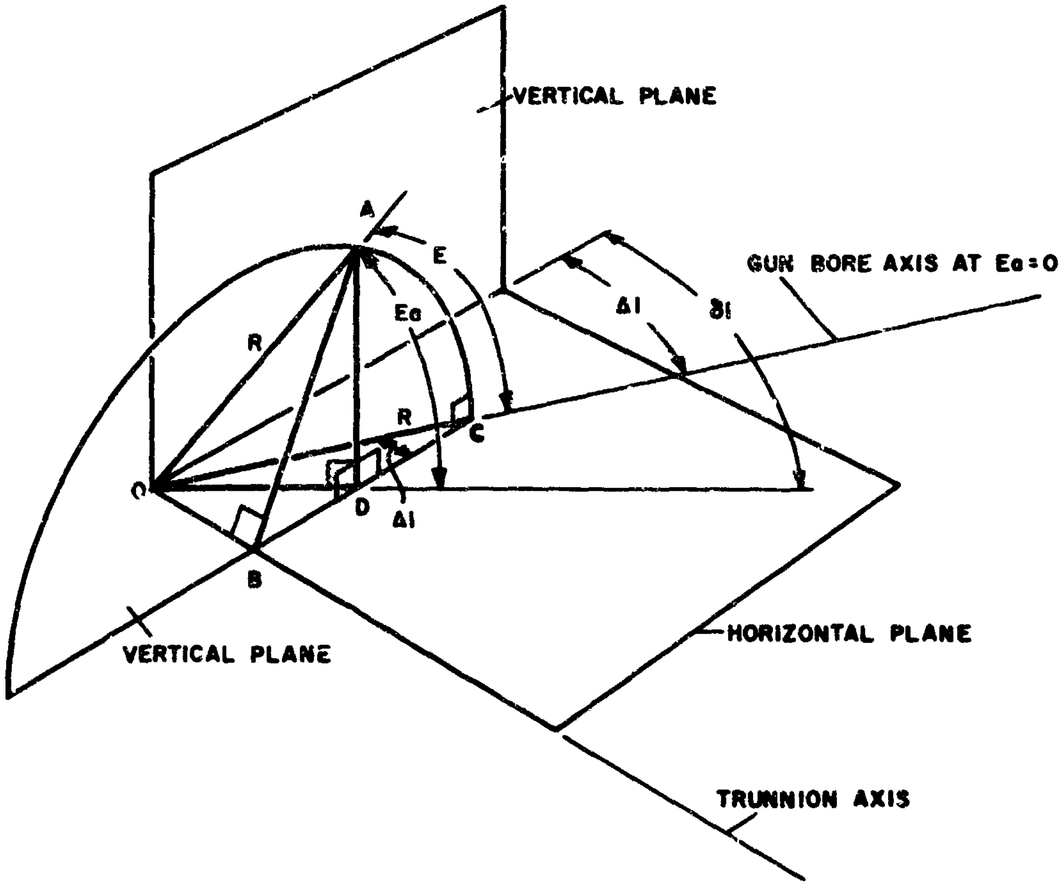


Figure C3. *Derivation of Equations 45, 46, and 47*

From right triangle ODB:

$$\cos \delta 1 = \frac{\overline{BD}}{\overline{OD}} \qquad \text{(Eq. C12)}$$

and from right triangle OAD:

$$\overline{OD}^2 = 1 - \overline{AD}^2 \qquad \text{(Eq. C13)}$$

but from right triangle ABD:

$$\overline{AD} = \overline{AB} \sin E \qquad \text{(Eq. C14)}$$

Comparing Equation C14 with Equation C3:

$$\overline{AD} = \sin Ea \qquad \text{(Eq. C15)}$$

Substituting Equation C15 in Equation C13:

$$\overline{OD}^2 = 1 - \sin^2 Ea = \cos^2 Ea$$

or

$$\overline{OD} = \cos Ea \qquad \text{(Eq. C16)}$$

Therefore, substituting Equations C10 and C16 in Equation C12:

$$\cos \delta 1 = \frac{\cos \Delta 1 \cos E}{\cos Ea} \qquad \text{(Eq. C17)}$$

Equation C17 is identical to Equation 47.

d. Sample Problem.

If the gun bore and trunnion axes were actually at an angle of 0.4 mil less than 90 degrees and the trunnions were rotated through an angle of 45 degrees, then in Equation C7:

$$\Delta 1 = 0.4 \text{ mil}$$
$$E = 45 \text{ degrees}$$

and

$$\sin Ea = \cos (0.4 \text{ mil}) \sin (45 \text{ degrees})$$
$$\sin Ea = (0.99999992)\ (0.707)$$
$$\sin Ea = 0.70699994$$

therefore:

$$Ea = 44°59'\ 28.81''$$

also, from Equation C17:

$$\cos \delta 1 = \frac{\cos (0.4 \text{ mil}) \cos (45 \text{ degrees})}{\cos (44°59'\ 28.81'')}$$

$$\cos \delta 1 = \frac{0.99999992\ (0.707)}{0.70720554}$$

$$\cos \delta 1 = 0.99970928$$
$$\delta 1 = 1°22'\ 6.22''$$

or about 24.3 mils.

e. An Approximation for Equation 22.

Referring to Figure 25, it can be seen that the following assumptions are justified:

1. The displacement component in the elevating plane of the aiming device, $Ph \cos A$, is small enough in comparison to $R \cos E$ that it may be neglected.

2. For an angle as small as Phd, it can be said that the tangent is equal to the angle in radians. Multiplied by the appropriate constant, the tangent, then, will equal the angle in degrees. ($K \tan Phd = Phd$)

Therefore, with negligible loss in accuracy, Equation 22 ($\tan Phd = \frac{Ph \sin A}{R \cos E - Ph \cos A}$) can be modified to the form:

$$Phd = \frac{KPh \sin A}{R \cos E} \qquad \text{(Eq. C18)}$$

APPENDIX D

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APPENDIX E

Glossary

aiming device. Any instrument, optical or electronic, used to establish target location data and apply laying information to a weapon.

aiming point. An object or point on which the sight of weapon is laid for direction, or on which an observer orients his observing instrument.

aiming post. A stake installed at a known position with respect to a weapon, for use as an aiming reference in indirect fire (equipped with lamp for night use).

angle of site. The vertical angle between the line of site and the horizontal plane.

axis. Imaginary or real line that passes through an object and about which the object turns or seems to turn. The centerline of the bore of a gun is its axis.

axis of sighting. Line taken through the sights of a gun, or through the optical center and centers of curvature of lenses in any telescopic instrument.

ballistic conditions. Conditions which affect the motion of a projectile in the bore and through the atmosphere, including muzzle velocity, weight of projectile, size and shape of projectile, rotation of the earth, density of the air, elasticity of the air and the wind.

base of trajectory. The straight line from origin to the level point (Figure 12).

boresighting. Process by which the axis of a gun bore and the axis of sighting of a gunsight are made parallel (infinity-boresighting) or are made to converge on a point (specific-range boresighting).

cant. Leaning or tilting to one side of any object; especially, the sidewise tilting of a gun, carriage mount or both.

cant angle. Vertical angle between a cross axis or elevating axis and the horizontal plane.

clinometer. Instrument for measuring or indicating angle of slope and elevation.

deck plane. The plane described as the trunnion axis rotates in azimuth.

deflection error. Distance to the right or left between the point aimed at and the burst of a projectile or the mean point of impact of a salvo.

direct fire. Engaging a target that is in the line of site of the weapon and the sighting system.

director. Electromechanical equipment used to track a moving target in azimuth and angular height and which, with the addition of other necessary information from an outside source, such as a radar set or a range finder, continuously computes firing data and transmits them to the guns.

emplacement. Prepared position for one or more weapons or pieces of equipment, for protection against hostile fire or bombardment, and from which they can execute their missions. Act of fixing a gun in a prepared position from which it may be fired.

gun elevation. Angle of the weapon bore with respect to the deck plane. The angle is measured in a plane (elevating plane) perpendicular to the deck (Figures 14 and 17).

indirect fire. Engaging a target that does not lie directly in the line of site or is not visible from the position of the weapon.

leveling. Adjusting any device, especially a gun and mount or sighting equipment, so that all horizontal or vertical angles will be measured in the true horizontal and vertical planes.

level point. The point on the descending branch of the trajectory that is at the same altitude as the origin (Figure 13).

line of elevation. The extension of the bore axis when the weapon is laid (Figure 12).

line of site. The axis of the aiming device extended to the target (Figure 12).

logistics. That part of the entire military activity which deals with production, procurement, storage, transportation, distribution, maintenance, and evacuation of personnel, supplies, and equipment; with induction, classification, assignment, welfare and separation of personnel; and with facilities required for the support of the military establishment including construction and operation thereof. It comprises both planning and implementation.

maximum elevation. The greatest vertical angle at which an artillery piece can be laid. It is limited by the mechanical structure of the piece.

mil (artillery mil). A unit of angular measurement used in military calculations. It is 1/6400th of a complete circle, or very nearly the angle between two lines which will enclose a distance of one yard at a range of 1000 yards. 17.78 mils are approximately equal to 1 degree. NOTE: The Navy mil and the French infantry mil are exactly one yard at a range of 1000 yards. There are 6283.1853 Navy or French infantry mils in a complete circle.

mount. Structure that supports any apparatus. A gun, a searchlight, a telescope, or a surveying instrument may have a mount.

null. Position or condition of reception of minimum signal.

off-carriage fire control. Process of controlling fire on a target with the aid of a sighting device which is not mounted on the weapon.

on-carriage fire control. Process of controlling fire on a target with the aid of a sighting device mounted on the weapon.

orthogonal. Property of being at right angles; or more generally, independent. Examples: the X, Y, and Z directions, or the R and θ direction in polar coordinates are orthogonal.

panoramic telescope. Telescope sight for artillery pieces that gives the gunner a wide field of view. It may be rotated so as to permit sighting in any direction without requiring the observer to change his position.

parallax. Apparent differences in the position of an object viewed from two different points, especially from a gun position and a directing point.

pitch angle. The angular deviation of the weapon fore-and-aft axis from the horizontal.

quadrant elevation. The vertical angle at the origin formed by the line of elevation and the base of the trajectory. It is the algebraic sum of the angle of elevation (superelevation) and the angle of site (Figure 12).

reticle. Measuring scale or mark placed in the focus of an optical instrument, used to determine the size, distance, direction, or position of objects. Generally, the reticles are etched on glass and the whole disk is spoken of as the reticle. Reticles are used on sighting telescopes and other fire control instruments.

sight. Mechanical or optical device for aiming a firearm or for laying a gun in position. It is based on the principle that two points in fixed relation to each other may be brought in line with a third. Sights are classified as fixed or adjustable depending on the provision made for setting windage and range, and also according to type. Glass sights comprise all sights which include an optical element, such as a collimator, telescope, periscope, etc.

superelevation. The ballistic correction for the drop of the projectile caused by gravity.

tracker. An instrument equipped with telescopes, used to continuously observe the present position of a moving target. Employed with a radar which is used for slant range measurement, the tracker furnishes the elements or present position data required by the computer. One of the components of a director.

trajectory. Path of a projectile, missile, or bomb in flight.

traverse. Movement of a gun on its mount, clockwise or counterclockwise, measured as an angle.

traverse axis. The axis perpendicular to the deck plane about which a weapon is turned to adjust its aim in azimuth.

trunnion. One of the two pivots supporting a piece of artillery on its carriage and forming the axis (trunnion axis), parallel to the deck plane, about which the barrel rotates when it is elevated. One of the two supporting pivots for holding an instrument on its mount.

weapon. The portion of an equipment or system that traverses and elevates to the direction of fire (as used in this Handbook).

weapon bore axis. The geometric center of the cylinder forming the bore of a weapon. The centerline about which the weapon tube is turned when the bore is being machined. The weapon bore axis is perpendicular to the trunnion axis and is parallel to the deck plane when gun elevation (Eg) is at zero.

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